Numerical Analysis and Computational Fluid Dynamics: Workshop in Honor of Münevver Tezer-Sezgin's 67th Birthday

Abstracts and Workshop Program

Department of Mathematics Middle East Technical University

19-20 April 2019

Preface

On the occasion of Prof. Tezer-Sezgin's 67th birthday and retirement, this workshop honors the outstanding achievements and valuable service to numerical analysis and computational fluid dynamics which Prof. Tezer-Sezgin has made over a career at the Department of Mathematics spanning more than 40 years.

The workshop features scientific contributions mainly by the former Ph.D. students of Prof. Tezer-Sezgin in the area of numerical analysis with applications specifically in computational fluid dynamics.

Content

Workshop Program7Invited Speakers8Nevzat Güneri Gençer8Bülent Karasözen9Kemal Leblebicioğlu10Hakan I. Tarman11Gamze Yüksel12	Preface	3
Invited Speakers8Nevzat Güneri Gençer8Bülent Karasözen9Kemal Leblebicioğlu10Hakan I. Tarman11Gamze Yüksel12Contributed Short Talks13Nagehan Alsoy Akgün13Cemre Aydın14Ali Deliceoğlu15Sevin Gümgüm16Merve Gürbüz17Selçuk Han Aydın18Gülnihal Meral19	Organizing Committee	6
Nevzat Güneri Gençer8Bülent Karasözen9Kemal Leblebicioğlu10Hakan I. Tarman11Gamze Yüksel12Contributed Short Talks13Nagehan Alsoy Akgün13Cemre Aydın14Ali Deliceoğlu15Sevin Gümgüm16Merve Gürbüz17Selçuk Han Aydın18Gülnihal Meral19	Workshop Program	7
Bülent Karasözen9Kemal Leblebicioğlu10Hakan I. Tarman11Gamze Yüksel12Contributed Short Talks13Nagehan Alsoy Akgün13Cemre Aydın14Ali Deliceoğlu15Sevin Gümgüm16Merve Gürbüz17Selçuk Han Aydın18Gülnihal Meral19	Invited Speakers	8
Kemal Leblebicioğlu10Hakan I. Tarman11Gamze Yüksel12Contributed Short Talks13Nagehan Alsoy Akgün13Cemre Aydın14Ali Deliceoğlu15Sevin Gümgüm16Merve Gürbüz17Selçuk Han Aydın18Gülnihal Meral19	Nevzat Güneri Gençer	8
Hakan I. Tarman11Gamze Yüksel12Contributed Short Talks13Nagehan Alsoy Akgün13Cemre Aydın14Ali Deliceoğlu15Sevin Gümgüm16Merve Gürbüz17Selçuk Han Aydın18Gülnihal Meral19	Bülent Karasözen	9
Gamze Yüksel12Contributed Short Talks13Nagehan Alsoy Akgün13Cemre Aydın14Ali Deliceoğlu15Sevin Gümgüm16Merve Gürbüz17Selçuk Han Aydın18Gülnihal Meral19	Kemal Leblebicioğlu	10
Contributed Short Talks13Nagehan Alsoy Akgün13Cemre Aydın14Ali Deliceoğlu15Sevin Gümgüm16Merve Gürbüz17Selçuk Han Aydın18Gülnihal Meral19	Hakan I. Tarman	11
Nagehan Alsoy Akgün13Cemre Aydın14Ali Deliceoğlu15Sevin Gümgüm16Merve Gürbüz17Selçuk Han Aydın18Gülnihal Meral19	Gamze Yüksel	12
Cemre Aydın14Ali Deliceoğlu15Sevin Gümgüm16Merve Gürbüz17Selçuk Han Aydın18Gülnihal Meral19	Contributed Short Talks	13
Ali Deliceoğlu15Sevin Gümgüm16Merve Gürbüz17Selçuk Han Aydın18Gülnihal Meral19	Nagehan Alsoy Akgün	13
Sevin Gümgüm16Merve Gürbüz17Selçuk Han Aydın18Gülnihal Meral19	Cemre Aydın	14
Merve Gürbüz17Selçuk Han Aydın18Gülnihal Meral19	Ali Deliceoğlu	15
Selçuk Han Aydın18Gülnihal Meral19	Sevin Gümgüm	16
Gülnihal Meral 19	Merve Gürbüz	17
	Selçuk Han Aydın	18
Pelin Şenel 20	Gülnihal Meral	19
	Pelin Şenel	20

Organizing Committee

Canan Bozkaya	Middle East Technical University
Ömer Küçüksakallı	Middle East Technical University
Önder Türk	Gebze Technical University

Workshop PROGRAM

Friday, April 19, 2019

09:00-9:40	Welcome and Opening Talks	
Chair: Önder Türk		
9:40-10:25	Hakan I. TARMAN : An Exploration of Numerical Approaches to Boltzmann Equation Regarding Hydrodynamics.	
10:25-10:45	Merve GÜRBÜZ : RBF Solution of MHD Convection Flow in a Lid-driven Cavity with an Obstacle.	
10:45-11:05	<u>Gülnihal MERAL</u> : On the DRBEM-FDM Solution of a Cancer Invasion Model.	
11:05-11:30	Coffee Break	
Chair: Gülnihal M	leral	
11:30-11:50	Sevin GÜMGÜM : Legendre Wavelet Solution of High Order Nonlinear Ordinary Delay Differential Equations.	
11:50-12:10	Nagehan ALSOY AKGÜN : DRBEM Solution of Nanofluids under DDMC in a Lid Driven Cavity.	
12:10-12:30	Selçuk Han AYDIN : Accuracy of the Different Element Types for the BEM Solutions of the 2- D Boundary Value Problems.	
12:30-14:00	Lunch Break	
Chair: Canan Bozkaya		
14:00-14:45	Gamze YÜKSEL : The finite element solutions of the Simplified MagnetoHydroDynamics Equations with time step methods and the effects of differential time filter on the solutions.	
14:45-15:30	Bülent KARASÖZEN: Structure Preserving Model Order Reduction.	
16:00-17:30	Cocktail/Retirement Party in Gündüz İkeda Seminar Room	
19:00	Workshop/Retirement Dinner	

Saturday, April 20, 2019

Chair: Önder Türk		
9:30-10:15	Nevzat Güneri GENÇER: Medical Electro-Thermal Imaging.	
10:15-10:35	Ali DELİCEOĞLU : Internal Flow Bifurcation in a T-shaped Cavity.	
10:35-10:55	Pelin SENEL : DRBEM Solution to MHD Flow Between Cylindrical Pipes with Slipping Walls.	
10:55-11:25	Coffee Break	
Chair: Nevzat Güneri Gençer		
11:25-12:10	Kemal LEBLEBİCİOĞLU: Unmanned Air, Sea and Underwater Vehicles	
12:10-12:30	<u>Cemre AYDIN</u> : DRBEM Solution of the Cauchy MHD Duct Flow with a Slipping Perturbed Boundary	
12:30-12:40	Workshop Closing	

Medical Electro-Thermal Imaging

Nevzat G. Gençer

Abstract

A hybrid system is proposed through the simultaneous utilization of thermal and electrical impedance imaging methods. The innovation of the approach relies on the frequency dependence of the tissue's electrical impedance. This facilitates the acquisition of multiple thermal images using medical infrared cameras with currents at different frequencies injected to the region of the body under inspection. The applied current and metabolic heat sources determine the temperature distribution on the body surface. The electrical currents increase the thermal contrast depending on the electrical properties of the tissues at the operation frequency. Consequently, the technique provides frequency dependent conductivity distribution data through thermal imaging which can be used as a basis for the detection of the breast carcinoma. Experimental aspects of this hybrid imaging modality are investigated using infrared cameras by applying sinusoidal currents in the 10 kHz-1 MHz frequency range to body phantoms. When the current injection frequency is increased to 1 MHz the image contrast is improved by 80%. Numerical simulations show that the depth-dependent imaging performance improves from 3 mm to 9 mm for a 1.5 mm tumor. The sensitivity of the technique can be further increased by an infrared camera with dual band imaging capability. The proposed approach has a potential to improve the sensitivity and accuracy of medical imaging over the standard thermography.

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Structure Preserving Model Order Reduction

B. Karasözen

Abstract

Discretization techniques for partial differential equations (PDEs) frequently lead to very high-dimensional numerical models with corresponding high demands concerning hardware and computation times. These high computational costs pose a serious problem in the context of multiquery and real-time computing scenarios. Such situations can be observed in the case in parameter studies, design, optimization, inverse problems or statistical analysis. Real-time scenarios consist of problems, where the simulation result is required very fast. In the past two decades efficient reduced order methods (ROMs) are developed to tackle these problems. In this talk we report about the structure preserving ROMs for conservative/dissipative PDEs; the nonlinear Schrödinger equation [2], Allen Cahn equation [4], Fitz-Hugh Nagumo equation [3], Ginzburg-Landau equation, Swift-Hohenberg equation [1] and shallow water equation. Current challenges in reduced order modelling like travelling waves, pattern formation, parametric ROMs and machine learning will be also discussed.

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Unmanned Air, Sea and Underwater Vehicles

Kemal Leblebicioğlu

Abstract

Unmanned air vehicles (UAV), unmanned sea vehicles (USV) and unmanned underwater vehicles (UUV) have become very popular in the last 10 years. The aim of this talk is to introduce various existing UAV, USV and UUV types and their features. The particular emphasis will be on the vehicles especially designed by myself and my research group. There will be a short discussion on how those vehicles can actually be built. Finally, some new concept vehicle designs will be mentioned.

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An Exploration of Numerical Approaches to Boltzmann Equation Regarding Hydrodynamics

Hakan I. Tarman

Abstract

The Lattice Boltzmann Method (LBM) has become an alternative tool for computational fluid dynamics (CFD). While traditional CFD methods are based on Navier-Stokes equations that describe the fluid in terms of macroscopic quantities, LBM takes a mesoscopic description of the fluid thus closing the gap between macroscale and microscale. Overall, LBM provides a simple and efficient framework for simulation of fluid flows. In this approach, Boltzmann kinetic equation with BGK collision operator is discretized over a square lattice and solved to compute the evolution of a particle distribution function whose velocity moments are connected to the macroscopic primitive variables such as velocity and density. In this talk, we present two main approaches in the velocity discretization of the Boltzmann equation, namely, Galerkin and Collocation approaches. The foundations leading to these approaches are systematically laid down and some numerical examples are presented.

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The finite element solutions of the simplified MagnetoHydroDynamics equations with time step methods and the effects of differential time filter on the solutions

G. Yüksel

Abstract

Magnetohydrodynamics (MHD) consider the dynamics of electrically conducting fluids and described by a set of equations which are a combination of the Navier-Stokes equations of fluids dynamics and Maxwells equations of electromagnetism. When the magnetic reunolds number $Rm \ll 1$, then the magnetic field associated with induced currents, $J \approx \sigma u \times B$ is negligible by comparison with the imposed magnetic field, thus MHD reduces to the simplified MHD (SMHD). Here, $Rm = uL/\mu > 0$; u is the characteristic speed, L is the length of the problem, $\mu > 0$ is the magnetic diffusivity, B is an applied (and known) magnetic field. There are three distinct cases for $Rm \ll 1$. The first, the imposed magnetic field is static, the flow is induced by some external agency. The second, the imposed magnetic field travels or rotates uniformly and slowly. The third, the imposed magnetic field oscillates extremely rapidly [1].

In this study, the Crank-Nicolson (CN) [2] and Backward-Euler (BE) [3] finite element solutions of SMHD are considered with numerical analyses, at first. Then, the differential filter term $\kappa(u-\bar{u})$ is added to SMHD equations to obtain SMHD Linear Time Relaxation Model (SMHDLTRM) [4]. The model is discretized by both CN and BE methods to obtain finite element solutions and numerical analysis are also proved for two methods [5].

Finally, in which cases, to may get more accurate solutions of SMHD and SMHDLTRM by using BE and CN methods are compared comprehensively. All computations are conducted by using FreeFem++ programme.

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DRBEM Solution of Nanofluids under DDMC in a Lid Driven Cavity

N. Alsoy-Akgün

Abstract

In this study the advantages of nanofluids on double diffusive mixed convection (DDMC) in a lid-driven cavity is analyzed by solving the velocity-vorticity form of the governing equations along with the energy and concentration equations. Numerical computations are conducted using the dual reciprocity boundary element method (DRBEM). Vorticity transport, energy and concentration equations are transformed to the form of modified Helmholtz equations by discretizing the time derivative terms first. The effects of Reynolds number (Re), Richardson numbers (Ri) and buoyancy ratio (N) for variation in volume fraction from 0 to 0.05 is presented for copper based nanofluid graphically and obtained results are good agreement with the results in [1].

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DRBEM Solution of the Cauchy MHD Duct Flow with a Slipping Perturbed Boundary

C. Aydin, M. Tezer-Sezgin

Abstract

In this study, the MHD flow problem is solved as a direct and a Cauchy problem in a rectangular duct with a perturbed, curved, and slip upper boundary. The aim is to recompute the slip length and the slipping velocity by employing the asymptotic analysis with respect to the perturbation parameter ϵ and solving MHD flow equations for the first order and the corrector solutions in the problem region. Hence, we are able to obtain the solution of MHD flow in the duct with curved perturbed boundary without discretizing the curved part of the boundary. The dual reciprocity boundary element method (DRBEM) is utilized for solving the coupled MHD equations in one stroke. The discretization of Cauchy problems results in ill-conditioned systems of linear algebraic equations, hence a regularization technique is needed. In this study, the Tikhonov regularization is used to obtain the solution of the DRBEM is discretizing only the boundary and providing both the velocity and its normal derivative values on the boundary. This enables us to recover the slip length on the perturbed boundary through the slip boundary condition.

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Internal Flow Bifurcation in a T-shaped Cavity

A. Deliceoğlu, E. Çelik, D. Bozkurt

Abstract

Flow development and eddy structure in a T-shaped cavity with lids moving in the same directions have been investigated using both tools from low-dimensional nonlinear dynamics and standard Galerkin finite element method. The homotopy invariance of the index is used to obtain the normal form of the stream function. The control space diagram is constructed for exhibiting the mechanism by which new vortex are obtained in the T-shaped cavity.

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Legendre Wavelet Solution of High Order Nonlinear Ordinary Delay Differential Equations

Sevin Gümgüm, Demet Ersoy Özdek, Gökçe Özaltun

Abstract

The purpose of this study is to illustrate the use of the Legendre wavelet method in the solution of high order nonlinear ordinary differential equations with variable and proportional delays. The main advantage of using Legendre polynomials lies in the orthonormality property, which enables a decrease in the computational cost and runtime. The method is applied to five differential equations up to sixth order, and the results are compared with the exact solutions and other numerical solutions when available. The accuracy of the method is presented in terms of absolute errors. The numerical results demonstrate that the method is accurate, effectual and simple to apply.

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RBF Solution of MHD Convection Flow in a Lid-driven Cavity with an Obstacle

Merve Gürbüz, M. Tezer-Sezgin

Abstract

In this study, steady, laminar, fully developed flow of a viscous, incompressible, electrically conducting fluid is considered in a lid-driven cavity with a square obstacle under the influence of uniform magnetic field. The MHD convection equations are solved in terms of stream function, vorticity and temperature by using the radial basis function (RBF) approximation. The numerical results are obtained for several values of Grashof number (Gr) and Hartmann number (M) at a fixed Prandtl number (Pr = 0.71) and Reynolds number (Re = 100) with Eckert number (Ec = 0 or Ec = 1) to analyze the effect of buoyancy force, magnetic field and viscous dissipation on the flow and heat transfer. It is found that secondary flow occurs at the left part of the cavity as Gr increases. Viscous dissipation retards the effect of magnetic field on the isotherms.

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Accuracy of the Different Element Types for the BEM Solutions of the 2-D Boundary Value Problems

S.H. Aydın

Abstract

In the Boundary Element Method (BEM) formulation of the 2-dimensional (2-D) boundary value problems, the boundary of the problem domain is separated in to elements. In this study, the well known element types constant, linear and quadratic elements, and also originally defined corresponding curved ones (curved constant, curved linear and curved quadratic) elements are used and compared in the solution of the Laplace equation on the both circular and elliptic domains. The comparison is done in terms of the L_2 norms, absolute errors and relative errors. In contrast to expectation, the linear elements give more accurate results compared to the quadratic ones. Therefore, we conclude that higher order elements always do not give higher accuracy in the numerical solutions due to the numerical errors.

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On the DRBEM-FDM Solution of a Cancer Invasion Model

<u>Gülnihal Meral</u>

Abstract

Cancer invasion is a key factor for metastasis which is defined as the formation of a secondary tumour at distal sites. The directed movement of cancer cells is most often influenced by two mechanisms: chemotaxis and haptotaxis. The former defines the cell motion in response to the concentration of a substance called chemoattractant (or chemorepellent). However, such gradients may lack in the solution; instead, adhesive molecules could be present in increasing amount along the extracellular matrix (ECM). Since cells have to adhere to the ECM fibres in order to be able to move, they migrate from a region of low concentration of those adhesive molecules to an area with a higher concentration. This is what is called haptotaxis. In this talk, the invasion model focusing on the effect of chemotaxis and haptotaxis [1] is considered which involves the interactions between the cancer cells, the normal cells and the matrix degrading enzyme (MDE) that is secreted by the cancer cells to invade the surrounding tissue. The corresponding model consists of a system of nonlinear reaction-diffusion-transport equations and difficulties arise in the numerical solution, especially in two-dimensional case, due to the nonlinear terms in the corresponding model. The model is solved by using a combined application of finite difference and dual reciprocity boundary element methods [2]. Effects of haptotaxis, chemotaxis, proliferation of cells are analyzed using numerical solutions. The boundary only nature of DRBEM gives the advantage of obtaining consistent solution with the real phenomena using less number of discretization methods comparing with the other domain discretization methods.

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DRBEM Solution to MHD Flow Between Cylindrical Pipes with Slipping Walls

P. Senel, M. Tezer-Sezgin

Abstract

In this study, MHD flow between concentric cylindrical pipes with arbitrarily conducting, slipping walls is investigated. A pressure driven, steady, laminar, fully developed flow of an electrically conducting, viscous fluid subjected to a horizontally applied uniform magnetic field is considered within the cross-section of the pipes. The non-dimensional form of the 2D coupled MHD equations are discretized by the Dual Reciprocity Boundary Element Method (DRBEM) with constant elements. The resultant matrix-vector equations for the velocity, induced magnetic field and their normal derivatives are combined and solved as a whole reducing the computational cost. Numerical results are presented for different slip lengths and wall conductance ratios at the inner and the outer boundaries. The influences of the slip, wall conductivity, magnetic field strength and the radius of the inner pipe are discussed. The obtained results show that, in the absence of the slip, boundary layers are developed close to the pipe walls and the maximum velocity occurs below and above the inner pipe. The main effect of increases in the magnetic field strength or inner pipe radius is the retardation of the flow. When both walls are slipping, the slip at the inner wall dominates the flow behavior and uniform velocity occurs at the center of the flow region. Increasing the wall conductivity enlarges the induced current loops next to the wall and the slip at the same wall retards this effect.

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