The finite element solutions of the Simplified MagnetoHydroDynamics Equations with time step methods and the effects of differential time filter on the solutions

G. Yuksel

Abstract

Magnetohydrodynamics (MHD) consider the dynamics of electrically conducting fluids and described by a set of equations which are a combination of the Navier–Stokes equations of fluid dynamics and Maxwell's equations of electromagnetism. When the magnetic reynolds number $R_m \ll 1$, then the magnetic field associated with induced currents, $J \sim \sigma u \times B$ is negligible by comparison with the imposed magnetic field, thus MHD reduces to the simplified MHD (SMHD). Here, $R_m = uL/\eta > 0$, u is the characteristic speed, L is the length of the problem, $\eta > 0$ is the magnetic diffusivity, B is an applied (and known) magnetic field. There are three distinct cases for $R_m \ll 1$. The first, the imposed magnetic field is static, the flow is induced by some external agency. The second, the imposed magnetic field travels or rotates uniformly and slowly. The third,the imposed magnetic field oscillates extremely rapidly [1].

In this study, the Crank-Nicolson (CN) [2] and Backward-Euler (BE) [3] finite element solutions of SMHD are considered with numerical analyses, at first. Then, the differential filter term $\kappa(u-\overline{u})$ is added to SMHD equations to obtain SMHD Linear Time Relaxation Model (SMHDLTRM) [4]. The model is discretized by both CN and BE methods to obtain finite element solutions and numerical analysis are also proved for two methods [5].

Finally, in which cases, to may get more accurate solutions of SMHD and SMHDLTRM by using BE and CN methods are compared comprehensively. All computations are conducted by using FreeFem++ programme.

References

[1] P.A. Davidson. An Introduction to Magnetohydrodynamics. Cambridge University Press, United Kingdom, 2001.

[2] G. Yuksel and R. Ingram, Numerical analysis of a finite element, Crank-Nicolson discretization for MHD flow at small magnetic Reynolds number, Int.J.Num.Anal.Model.10 (2013), no.1, 74-98.

[3] G. Yuksel, O.R. Isik, Numerical analysis of Backward-Euler discretization for simplified magneto-hydrodynamic flows, Appl. Math. Modell.,39 (7) (2015),1889-1898.

[4] O. R.,Isik, Spin up problem and accelerating convergence to steady state, Appl. Math. Model. 37 (5) (2013): 3242-3253.

[5] W.Layton, Introduction to the Numerical Analysis of Incompressible Viscous Flows, SIAM, Philadelphia, 2008.

Correspondance

ngamze@mu.edu.tr; Faculty of Science Department of Mathematics, Mugla Sitki Kocman University, 48000, Mugla, Turkey