# M E T U
# Department of Mathematics

| | Introduction to Differential Equations<br>Final Exam | |
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| Code : *Math 219*<br>Semester : *Fall 2018*<br>Date : *10 January 2019*<br>Time : *17:00*<br>Duration : *140 minutes* | Last Name : Name :<br>Department : Student No. :<br>Signature :<br>7 QUESTIONS ON 4 PAGES<br>TOTAL 100 POINTS |

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | **SHOW YOUR WORK** | |
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**Question 1 (10 pts)** Find all solutions of the differential equation

$$y' = \frac{y + xe^{-y/x}}{x}.$$

**Question 2 (10 pts)** Write a suitable form for a particular solution of the differential equation

$$y^{(4)} + 2y'' + y = t\cos(t) + te^{2t}$$

<u>with at most 6 coefficients</u>, if the method of undetermined coefficients is to be used. Do not evaluate the coefficients.

**Question 3 (20 pts)** Find all solutions of the following $2 \times 2$ non-homogeneous system by using variation of parameters:

$$\mathbf{x}' = \begin{bmatrix} 2 & -5 \\ 1 & -2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ \cos(t) \end{bmatrix}, \qquad 0 < t < \pi.$$

**Question 4 (10 pts)** Suppose that $\mathcal{L}\{g(t)\} = G(s)$. Express $\mathcal{L}^{-1}\left\{\dfrac{G(s)}{s^2+4}\right\}$ as a convolution integral in terms of $g(t)$.

**Question 5 (20 pts)** By using the Laplace transform, solve the initial value problem

$$y'' + y = f(t) + \delta(t-3), \qquad y(0) = y'(0) = 0$$

for $t \geq 0$ where $\delta(t)$ denotes the impulse function and $f(t) = \begin{cases} 1, & 0 \leq t < 1, \\ -1, & 1 \leq t < 2, \\ 0, & 2 \leq t. \end{cases}$

**Question 6 (10 pts)** Suppose that $f(x) = \begin{cases} x, & 0 \leq x < 1, \\ 1, & 1 \leq x < 2. \end{cases}$ Sketch the graph of the odd extension of $f(x)$ as a periodic function with period 4, for $-6 \leq x \leq 6$. Find the Fourier series expansion of this odd extension.

**Question 7 (20 pts)** Find the solution of the heat equation below subject to the conditions:

$$\begin{aligned} u_{xx} &= u_t, \quad 0 < x < \pi, \ \ t > 0 \\ u_x(0,t) &= u_x(\pi,t) = 0, \qquad t > 0 \\ u(x,0) &= 1 - \cos(4x), \qquad 0 < x < \pi \end{aligned}$$

**Show all steps:** Apply separation of variables, solve the resulting two point boundary value problem, and find the solution $u(x,t)$.