M E T U
Department of Mathematics

	Introduction to Differential Equations **MidTerm 1**	
Code: *Math 219* Semester: *Spring 2019* Date: *6 April 2019* Time: *17:00* Duration: *120 minutes*	Last Name: Name: Department: Student No.: Signature:	
	4 QUESTIONS ON 4 PAGES TOTAL 100 POINTS	
1 | 2 | 3 | 4	**SHOW YOUR WORK**	

Question 1 (25 pts) Consider the differential equation

$$\left(\frac{\sin(y)}{y} - 2e^{-x}\sin(x)\right)dx + \left(\frac{\cos(y) + 2e^{-x}\cos(x)}{y}\right)dy = 0.$$

(a) Show that $\mu(x, y) = ye^x$ is an integrating factor by using the test for exactness.

(b) Find all solutions of the equation.

(c) Find the solution that satisfies $y(\pi) = \pi$.

Question 2 (25 pts) Consider the differential equation

$$\frac{ds}{dx} = 2x + 4x^3 e^{-s}.$$

(a) Make the substitution $y = e^s$ and show that the resulting equation is linear in y.

(b) Find the solution of the equation that satisfies $s(0) = \ln(2)$.

(c) Show that the interval $[0, \infty)$ is a subset of the domain of the solution in part (b).

Question 3 (25 pts) Find all solutions of the system $\mathbf{x}' = \begin{bmatrix} 1 & -4 \\ 5 & -3 \end{bmatrix} \mathbf{x} + \begin{bmatrix} e^{-t} \\ 2e^{-t} \end{bmatrix}$.

Question 4 (25 pts) Let $a \in \mathbb{R}$. Consider the system $\mathbf{x}' = \begin{bmatrix} -2 & 0 & 1 \\ 0 & -3 & 1 \\ 0 & a & -1 \end{bmatrix} \mathbf{x}$.

(a) Find the value of a such that the system has a constant solution $\mathbf{x} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$ different from $\mathbf{0}$. (Do not solve the system.)

(b) Find the value of a for which the coefficient matrix has only one eigenvalue. Find all solutions of the system for this value of a.