

M E T U
Department of Mathematics

Introduction to Differential Equations						
MidTerm 1						
Code : <i>Math 219</i>			Last Name :			
Acad. Year : <i>2017-2018</i>			Name :		Student No. :	
Semester : <i>Spring</i>			Department :		Section :	
Coordinator: <i>Özgür Kişisel</i>			Signature :			
Date : <i>March.31.2018</i>			6 QUESTIONS ON 4 PAGES TOTAL 100 POINTS			
Time : <i>17:00</i>						
Duration : <i>120 minutes</i>						
1	2	3	4	5	6	SHOW YOUR WORK

Question 1 (13 pts) Find an integrating factor of the form $\mu(y) = e^{ay}$ for the equation

$$-dx + (2ye^y + x)dy = 0$$

where a is a constant. Using this integrating factor, find all solutions of the equation.

Question 2 (12 pts) Verify that $y_1(t) = 1$ and $y_2(t) = (t - 1)^{5/4} + 1$ are both solutions of the initial value problem

$$\frac{dy}{dt} = \frac{5}{4}(y - 1)^{1/5}, \quad y(1) = 1.$$

Why does this not contradict the existence-uniqueness theorem?

Question 3 (25 pts) A ball with mass 0.5kg is thrown upward with an initial velocity 20m/s from the roof of a building 30m high. Assume that there is a force due to air resistance of $|v|/20$, where the velocity v is measured in m/s . Find the time at which the ball reaches the maximum height above the ground. ($g = 9.8\text{m/s}^2$.)

Question 4 (25 pts) Find the solution of the initial value problem

$$\mathbf{x}' = \begin{bmatrix} 1 & -4 \\ 5 & -3 \end{bmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{bmatrix} 2 \\ -3 \end{bmatrix}.$$

Question 5 (15 pts) Find all solutions of the system

$$\mathbf{x}' = \begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix} \mathbf{x}.$$

Question 6 (10 pts) Show that the following set of solutions of a certain system $\mathbf{x}' = A\mathbf{x}$ is linearly independent :

$$\mathbf{v}^{(1)} = \begin{bmatrix} e^t \\ -2e^t \\ 0 \end{bmatrix}, \quad \mathbf{v}^{(2)} = \begin{bmatrix} 2e^t \\ 0 \\ -4e^t \end{bmatrix}, \quad \mathbf{v}^{(3)} = \begin{bmatrix} -e^t \\ 4e^t \\ e^t \end{bmatrix}.$$