M E T U
Department of Mathematics

	Introduction to Differential Equations MidTerm 1	

Code : *Math 219* Acad. Year : *2016-2017* Semester : *Spring* Coordinator : *Özgür Kişisel* Date : *April.08.2017* Time : *17:00* Duration : *120 minutes*	Last Name : Name : Student No. : Department : Section : Signature : 4 QUESTIONS ON 4 PAGES TOTAL 100 POINTS

1	2	3	4	SHOW YOUR WORK	

Question 1 (9+9+7= 25 pts) (a) Find an integrating factor of the form $\mu(x, y) = f(x)e^x$ for the differential equation

$$(x + 2)\sin(y)dx + x\cos(y)dy = 0.$$

(b) Solve the differential equation.

(c) Is there a solution of the ODE of the form $y(x)$ such that $y(1) = \frac{\pi}{2}$? What does the existence-uniqueness theorem say for this initial condition?

Question 2 (25 pts) Suppose that the volume of a round ball decreases at a rate directly proportional to its radius (while its spherical shape is preserved) . If initially the ball has a radius of 10 units and after 2 minutes it has a radius of 6 units, then find the time at which the ball shrinks to a single point.

Question 3 (8+8+9=25 pts) (a) A certain linear system of algebraic equations in the variables $x_1, \ldots, x_5$ has the following augmented matrix after row reduction.

$$\left[\begin{array}{ccccc|c} 0 & 1 & 0 & 3 & 2 & 0 \\ 0 & 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right].$$

Write down all possible solutions of the system.

(b) Suppose that $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$ are two solutions of the 2×2 homogenous linear system $\mathbf{x}' = A\mathbf{x}$ where the coefficients of the matrix A are continuous. Suppose that

$$\mathbf{x}^{(1)}(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \mathbf{x}^{(2)}(0) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$

Show that $\{\mathbf{x}^{(1)}, \mathbf{x}^{(2)}\}$ is a fundamental set of solutions.

(c) Let

$$\mathbf{v}^{(1)} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \mathbf{v}^{(2)} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \mathbf{v}^{(3)} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \mathbf{v}^{(4)} = \begin{bmatrix} 1 \\ 4 \\ 9 \\ 16 \end{bmatrix}.$$

Suppose that A is a constant 4×4 matrix such that $A\mathbf{v}^{(1)} = 5\mathbf{v}^{(1)}, A\mathbf{v}^{(2)} = 5\mathbf{v}^{(2)} + \mathbf{v}^{(1)}, A\mathbf{v}^{(3)} = 5\mathbf{v}^{(3)}, A\mathbf{v}^{(4)} = -\mathbf{v}^{(4)}$. Find 4×4 matrices P, J such that P is invertible, J is in Jordan form and $AP = PJ$.

Question 4 (13+12=25 pts) Consider the following nonhomogenous 2×2 linear system:

$$\begin{aligned} x_1' &= -x_1 - 2x_2 + \frac{e^{-t}}{2} \\ x_2' &= 8x_1 - x_2 \end{aligned}$$

(a) Find a fundamental matrix for the associated homogenous system.

(b) Find a particular solution of the system by using variation of parameters.