

M E T U Department of Mathematics

Math 219 Introduction to Differential Equations Fall 2019 Midterm 1 9 November 2019 13:30		
FULL NAME	STUDENT ID	DURATION 120 MINUTES
5 QUESTIONS ON 4 PAGES	SHOW ALL YOUR WORK	TOTAL 100 POINTS

(25 pts) 1. A tank with a capacity of 500ℓ originally contains 200ℓ of water with $100g$ of salt in solution. Water containing $1g$ of salt per liter is entering at a rate of $3\ell/min$, and the mixture is allowed to flow out of the tank at a rate of $2\ell/min$.

(a) Find the amount of salt in the tank prior to the instant when the solution begins to overflow.

(b) Find the concentration (in grams per liter) of salt in the tank when it is at the point of overflowing.

(c) Compare this concentration with the theoretical limiting concentration if the tank had infinite capacity.

(15 pts) 2. Consider $(y - xy^3)dx + xdy = 0$, $y(1) = 1$. Find an integrating factor of the form $\mu(xy)$ and solve this initial value problem.

(15 pts) 3. $\frac{dy}{dx} = \frac{x^2}{1-y}$, $y(0) = 0$.

(a) Show, without solving, that this initial value problem has a unique solution on some interval $(-h, h)$.

(b) Solve the initial value problem and find the largest possible value of h .

(20 pts) 4. Consider the system

$$\mathbf{x}' = \begin{bmatrix} -1 & 1 \\ -1 & -3 \end{bmatrix} \mathbf{x}$$

(a) Find a fundamental matrix for this system.

(b) Find the solution of the system that satisfies the initial condition $\mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(25 pts) 5. Consider the system

$$\mathbf{x}' = \begin{bmatrix} 1 & a \\ 2 & b \end{bmatrix} \mathbf{x}$$

where $a, b \in \mathbb{R}$. Suppose that one of the eigenvalues of the coefficient matrix is $+5i$.

(a) Find a and b .

(b) Find a fundamental set of real valued solutions for this system.