M E T U
Department of Mathematics

	Introduction to Differential Equations MidTerm 1	

Code: *Math 219* Semester: *Fall 2018* Date: *17 November 2018* Time: *13:30* Duration: *120 minutes*	Last Name: Name: Department: Student No.: Signature: 6 QUESTIONS ON 4 PAGES TOTAL 100 POINTS

1	2	3	4	5	6	**SHOW YOUR WORK**	

Question 1 (20 pts) Consider the non-exact differential equation

$$\left(\frac{1}{xy} - 3x^2\right) dx + \frac{1}{y^2} dy = 0.$$

(a) Find an integrating factor of the form $\mu = x^n$.

(b) Find the solution curves of the given differential equation in the form $\Phi(x, y) = c$.

(c) For each of the following initial conditions, determine the interval in which the solution is valid: (i) $y(1) = 2$, (ii) $y(1) = -2$.

Question 2 (20 pts) A tank, with capacity 200ℓ (liters) contains 10ℓ of water in which is dissolved $40g$ (grams) of chemical. A solution containing $2g/\ell$ of the chemical flows into the tank at a rate of $4\ell/min$, and the well-stirred mixture flows out at a rate of $2\ell/min$. Determine the function $Q(t)$ describing the amount of the chemical in the tank at any time t before the tank overflows.

Question 3 (10 pts) Suppose that $A(t)$ is a 2×2 matrix whose entries are continuous functions and $\mathbf{g}(t) \neq \mathbf{0}$ is a 2×1 vector of continuous functions. Assume that $\mathbf{x}^{(1)}$, $\mathbf{x}^{(2)}$, $\mathbf{x}^{(3)}$ are solutions of the system $\mathbf{x}' = A(t)\mathbf{x} + \mathbf{g}(t)$. Furthermore, suppose that

$$\mathbf{x}^{(1)}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \ \mathbf{x}^{(2)}(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \ \mathbf{x}^{(3)}(0) = \begin{bmatrix} 3 \\ 3 \end{bmatrix}.$$

(a) Find an expression for the general solution of the system above, in terms of $\mathbf{x}^{(1)}$, $\mathbf{x}^{(2)}$, $\mathbf{x}^{(3)}$.

(b) Find the solution of the system that satisfies the initial condition $\mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ in terms of $\mathbf{x}^{(1)}$, $\mathbf{x}^{(2)}$, $\mathbf{x}^{(3)}$. is certain to exist.

Question 4 (15 pts) Solve the initial value problem $\mathbf{x}' = \begin{bmatrix} 1 & -5 \\ 1 & -3 \end{bmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ where $\mathbf{x} = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$.

Question 5 (10 pts) Determine (without solving the equation) an interval in which the solution of the initial value problem

$$y' + (\tan t)y = \sin t, \quad y(\pi) = 0$$

Question 6 (25 pts) Consider the system of ODE's

$$\mathbf{x}' = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ 5 & 0 & 0 \end{bmatrix} \mathbf{x}.$$

(a) Determine the eigenvalues and the corresponding eigenvectors, as well as generalized eigenvectors, of the coefficient matrix.

(b) Find the general solution of the given system.

(c) Determine all possible initial conditions $\mathbf{x}(0)$ such that for the solution $\mathbf{x}(t)$ of the corresponding initial value problem, $\lim_{t \to +\infty} \mathbf{x}(t)$ exists and is finite.