M E T U
Department of Mathematics

	Introduction to Differential Equations **MidTerm 1**	

Code : *Math 219* Acad. Year : *2017-2018* Semester : *Fall* Coordinator : *Özgür Kişisel* Date : *November.18.2017* Time : *13:30* Duration : *120 minutes*	Last Name : Name : Student No. : Department : Section : Signature : 6 QUESTIONS ON 4 PAGES TOTAL 100 POINTS

1	2	3	4	5	6	**SHOW YOUR WORK**	

Question 1 (8+12= 20 pts) Given the initial value problem (IVP)

$$ydx + (2xy - e^{-2y})dy = 0, \qquad y(-1) = 2$$

(a) By using existence-uniqueness theorem show that this IVP has a unique solution.

(b) By finding an integrating factor of the form $\mu(y)$ solve the IVP.

Question 2 (20 pts) Suppose that the population of whales in world's oceans increases by 20 % per year in the absence of external factors. If there are originally 1000 whales and scientists are allowed to capture 150 whales per year, derive an IVP that gives the number of whales present in any year and solve it.

Question 3 (10 pts) Solve the initial value problem

$$y' + 2(x+2)y^2 = 0, \qquad y(0) = -\frac{1}{12}$$

Question 4 (15 pts) Find a fundamental matrix for the system

$$\mathbf{x}' = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \mathbf{x}$$

Question 5 (10 pts) Show that if $\mathbf{v}$ is an eigenvector of matrix A, then $\mathbf{v}$ is also an eigenvector of A^3. Use this to find the general solution of the system $\mathbf{x}' = A^3\mathbf{x}$ if all solutions of the system $\mathbf{x}' = A\mathbf{x}$ are of the form

$$c_1 e^{-t} \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + c_2 e^{2t} \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + c_3 e^{-3t} \begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix}$$

where A is a 3×3 constant matrix and $c_1, c_2, c_3 \in \mathbb{R}$.

Question 6 (25 pts) Find the general solution of the system

$$\mathbf{x}' = \begin{bmatrix} 2 & -5 \\ 1 & -2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ \cos t \end{bmatrix}, \qquad 0 < t < \pi$$