MATH153 Exercise Sheet 3

1. Solve the inequality $|2x-1| < |x-4|$
   (a) algebraically
   (b) by interpreting the graphs on the xy-plane.

2. Compute the following limits if they exist. Do **NOT** use derivative or L'Hospital's Rule.
   (a) $\lim_{x\to\infty} \sqrt{x^9+x} - x^5$
   (b) $\lim_{x\to 1^-} \frac{\sqrt{1-x}}{1-\sqrt{x}}$
   (c) $\lim_{x\to -\infty} \frac{2x^3-x+7}{\sqrt{3x^6+x^5-4}}$
   (d) $\lim_{x\to\infty} \frac{2-cos^3(x)}{x+sin^4(x)}$
   (e) $\lim_{x\to 2} \frac{|x^3-2x^2+x-2|}{x^2-5x+6}$
   (f) $\lim_{x\to 3} \frac{|x^3-26|+|x^3-28|}{|1-x|+1}$
   (g) $\lim_{t\to 0} t^2 e^{sin(\frac{153}{t})}$
   (h) $\lim_{x\to 0} \frac{1-cosx}{x^2}$
   (i) $\lim_{x\to\infty} \frac{\lfloor x \rfloor}{x}$
   (j) $\lim_{x\to 0^+} x^3 cos(\frac{2}{x})$
   (k) $\lim_{x\to -\infty} (x+\sqrt{x^2+4x+1})$
   (l) $\lim_{x\to 0} \frac{tan(x)-sin(x)}{sin(x)+x}$
   (m) $\lim_{x\to 0} \frac{tan(x)}{x^3}$
   (n) $\lim_{x\to 0} \frac{\tan(3x)+\sin(x)}{3x}$
   (o) $\lim_{x\to 5} \frac{|x+5|}{|x|-5}$
   (p) $\lim_{x\to\infty} \frac{153\sin(x)}{x^2}$

3. State the $\epsilon-\delta$ definition of $\lim_{x\to a} f(x) = L, \lim_{x\to a} f(x) = \infty, \lim_{x\to\infty} f(x) = L$ and $\lim_{x\to\infty} f(x) = \infty$.

4. Using the graph of the function $y = f(x) = \begin{cases} 3 & \text{if } x \leq 2 \\ 2 - x & \text{if } x > 2 \end{cases}$,

   sketch the graph of $y = f(2 - x) - 1$.

5. Determine if the given statement is true or false. If it is true, **prove** or **explain** it. If it is false, **give** a counter example.

   (a) If $\lim_{x \to x_0} f(x) = 0$, then $\lim_{x \to x_0} \frac{1}{f(x)} = \infty$.

   (b) If $\lim_{x \to a} f(x) = 0$ and $\lim_{x \to a} g(x) = 0$, then $\lim_{x \to a} \frac{f(x)}{g(x)}$ does not exist.

   (c) If $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ do not exist, then $\lim_{x \to a} \frac{f(x)}{g(x)}$ does not exist.

   (d) If f is an odd function, $L \in \mathbb{R}$, and $\lim_{x \to c} f(x) = L$, then $\lim_{x \to -c} f(x) = -L$.

6. Let $f(x)$ be bounded function and $\lim_{x \to c} g(x) = 0$ Then $\lim_{x \to c} [f(x)g(x)] = 0$

7. using formal definition of limit show the following:

   (a) $\lim_{x \to 1} \frac{-2}{(x-1)^2} = -\infty$ by

   (b) $\lim_{x \to 1} \frac{x+5}{1+\sqrt{x}} = 3$

   (c) $\lim_{x \to 1} \frac{1}{x+1} = \frac{1}{2}$

   (d) $\lim_{x \to 1} (x^3 - 5x + 8) = 4$