MATH153 - Recitation 5

1. Find k such that the line $y = k - x$ is normal to the graph of $f(x) = x^2$ (Consider slope of tangent line of $f(x)$ at the point (a,b).) Recall: Let l_1 and l_2 be two lines and m_1 and m_2 be slopes of l_1, l_2, respectively. $m_1 m_2 = -1$ iff l_1 is perpendicular to l_2

2. Find the equation of the tangent line of the $y = \sqrt{x} + 1$ at the point $(1, 2)$ and tangent

3. Find the eqn of the tangent lines to the curve $y = x^3 + x$ which pass through the point $(2, 2)$

4. Find the derivative of the following functions using the definition of the derivative

   (a) $f(x) = 2x$

   (b) $f(x) = 2x^2 + 3x + 6$

   (c) $f(x) = \dfrac{x}{x+1}$

   (d) $f(x) = \sqrt{2x+3}$

5. Evaluate the following limits

6. Find derivative of the following functions

   (a) $f(x) = \sqrt{x} sinx cosx$

   (b) $f(x) = \dfrac{xsinx}{x^2+1}$

   (c) $f(x) = tan(\sqrt{x^3+3} sin(1+x^2))$

   (d) $f(x) = \dfrac{sin(\sqrt{x^2+2})}{cos(x^2+1)}$

   (e) $f(x) = \sqrt{cos^3(x^4+5x+1) + tan^2(x^4+5x+1)}$

7. (a) Find fourth derivative of $f(x) = 4\sqrt[5]{x^3} - \dfrac{1}{8x^2} - \sqrt{x}$

   (b) Find nth derivative of $f(x)$ where $f(x) = sin(2x) - cos(x)$

8. Prove the $(1+x)^r < 1 + rx$ provided that $0 < r < 1$ and $x > 0$

9. Let f be a differentiable function such that f' is continuous and $f(0) = 0$,$f(1) = 1$,$f(2) = 154$, and $f(3) = 153$. Show that the graph of $y = f(x)$ has a horizontal tangent line.

10. Suppose f is twice differentiable on an interval I(i.e. f'' exists on I). Suppose that the points 0 and 2 belong to I and that $f(0) = f(1) = 0$ and $f(2) = 1$. Prove that

(a) $f'(a) = \dfrac{1}{2}$ for some point $a \in I$.

(b) $f'(b) > \dfrac{1}{2}$ for some point $b \in I$.

(c) $f'(c) = \dfrac{1}{7}$ for some point $c \in I$.

11. Let f be a function such that $|f(x)| \leq x^2$ for all x. Prove that f is differentiable at $x = 0$, and find $f'(0)$.

12. Prove the following

    (a) If $g'(x) < 0$ on (a, b) then $g(a) > g(b)$ (Hint: Use MVT)

    (b) If $g'(x) < 0$ for all x then $g(x)$ is 1-1

    (c) If $f'(x) = g'(x)$ then $f(x) = g(x) + k$ for some k

13. Find dy/dx and y'' for each of the followings

    (a) $2xy + x^3 - 3y^2 = 5$

    (b) $2x^3 + x^2y + y^2 = 4$ at the point $(-1, 2)$

    (c) $x^3 - 2x^2 + y^4 = 8$

    (d) $x^3 + 2y^2 - xy = 2$ at the point $(0, -1)$