MATH153 -Exercise Sheet 2

Date: February 27

1. $$f(x) = \begin{cases} 3 & \text{if } x \leq 2 \\ 2 - x & \text{if } x > 2 \end{cases}$$

   By shifting and scaling the graph of $y = f(x)$ sketch the graphs of

   (a) $y = 2f(3x + 2) + 4$

   (b) $y = f(2 - x) - 1$.

2. Determine whether the following statements are true or false

   (a) if $2 < x < 6$, then $1 < |x - 3| < 3$

   (b) if $|2x - 1| < 5$ then $-3 < x < 3$

   (c) $\sin(4) > 0$

3. Given the function

$$f(x) = \begin{cases} \sin(x) & \text{if } x < \pi \\ 2 & \text{if } \pi \leq x < 5 \\ 27 - x^2 & \text{if } x \geq 5 \end{cases}$$

   compute the following limits.

   (a) $\lim_{x \to 0} f(x)$

   (b) $\lim_{x \to \pi} f(x)$

   (c) $\lim_{x \to 4} f(x)$

   (d) $\lim_{x \to 5} f(x)$

4. Evaluate each of the following limits if exist.

   (a) $\lim_{x \to -2} \dfrac{3x + 6}{|x + 2|}$ (Right left limit)

   (b) $\lim_{x \to 0} \dfrac{3(2x - 3)^2 - 27}{x}$

(c) $\lim_{x\to 4} \dfrac{x-\sqrt{3x+4}}{4-x}$

5. Determine whether the following statements are true or false

(a) if $f(c) = L$, then the limit of $\lim_{x\to c} f(x) = L$

(counterexample: $f(x) = \begin{cases} x & \text{if } x \leq 0 \\ 1 & \text{if } x = 0 \end{cases}$

(b) If the limit of $\lim_{x\to c} f(x) = L$ then $f(c) = L$

(c) If $\lim_{x\to a} f(x) = L$ then $\lim_{x\to a}(f(x) - L) = 0$

(d) If $\lim_{x\to a} f(x) = \infty$ and $\lim_{x\to a} g(x) = \infty$, then $\lim_{x\to a}(f(x) - g(x)) = 0$

6. Use the formal definition to prove the followings:

(a) $\lim_{x\to 1} \dfrac{1}{x+1} = \dfrac{1}{2}$

(b) $\lim_{x\to 1}(x^3 - 5x + 8) = 4$

(c) $\lim_{x\to 1} \dfrac{x+5}{1+\sqrt{x}} = 3$

(d) $\lim_{x\to 0} \dfrac{sin(x)}{x} = 1$