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METU MATH 116, Exam 2

Thursday, May 17, 2012, at 17:40 (90 minutes)

Instructors: Beyaz, Ercan, Pamuk, Solak

Instructions: It should be obvious to the grader how to read your solutions. Please work carefully.

Problem 1. (10pts) Assume that $R = \{\begin{bmatrix} m & 2n \\ n & m \end{bmatrix} \mid m, n \in \mathbb{Z}\}$ and $R' = \{m + n\sqrt{2} \mid m, n \in \mathbb{Z}\}$ are rings with respect to their usual operations. Prove that R and R' are isomorphic rings.

Problem 2. (10pts) Let $G = A_4$ the alternating group. Let $H = \{(1), (12)(34), (13)(24), (14)(23)\}$ be a normal subgroup of A_4.

(a) List all the elements of A_4.

(b) Write out the distinct elements of G/H.

(c) Make a multiplication table for G/H.

Problem 3. (10pts) Let R be a ring. The center of R is defined as $Z(R) = \{x \in R | xr = rx \ \forall r \in R\}$. Show that $Z(R)$ is a subring of R. Is $Z(R)$ an ideal of R? Why?

Problem 4. (15pts) Let $\sigma = (12)(58)(346)(52)(41)(37)(67)$ be an element of S_8

(a) Write σ as a product of disjoint cycles.

(b) Find the order $\circ(\sigma)$.

(c) Is $\sigma \in A_8$? Why?

(d) Find σ^{265}.

(e) Given

$$\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 & 4 & 6 & 2 & 3 & 7 & 8 & 1 \end{bmatrix}.$$

Find $\alpha \in S_8$ such that $\alpha\sigma\alpha^{-1} = \beta$.

Problem 5. (10pts) Let R be the set of all matrices of the form $R = \{\begin{bmatrix} a & -b \\ b & a \end{bmatrix} \mid a, b \in \mathbb{R}\}$. Assume R is a ring with respect to matrix addition and multiplication. Answer the following questions and in each of them prove your claim.

(a) Does R have a multiplicative identity? If so, find it.

(b) Is R an integral domain?

(c) Is R a field?