

Name:

Student number:

METU MATH 116, Midterm 2

Thursday, April 30, 2015, at 17:40 (100 minutes), totally 60 points

Instructions: Please, be accurate and show clearly the logic of your solutions. Only the answers are not enough: indicate your calculations and arguments.

Problem 1. (15 pts)

Let $f = (1234)(1356)(1457)$ be a permutation in S_7 .

| | |
|----------|--|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| Σ | |

(a) Write f as a product of disjoint cycles.

(b) Write f as a product of transpositions.

(c) Does f belong to the alternating group A_7 ? Explain your answer.

(d) Find gfg^{-1} where $g = (1367)$.

Problem 2. (15 pts) Let $H \subseteq S_4$ be a cyclic subgroup generated by (124) .

(a) Find the number of cosets of H in S_4 .

(b) List the elements of the coset $(13)H$.

(c) Is H a normal subgroup of S_4 ? Explain your answer.

Problem 3. (15 pts) Let G be the quaternion group $G = \{1, i, j, k, -1, -i, -j, -k\}$ where $i^2 = j^2 = k^2 = -1$, $ij = -ji = k$, $jk = -kj = i$, $ki = -ik = j$ and let G' be the Klein four group $G' = \{e, a, b, ab\}$ where $a^2 = b^2 = e$ and $ab = ba$. Define a function $\phi : G \rightarrow G'$ by $\phi(1) = \phi(-1) = e$, $\phi(i) = \phi(-i) = a$, $\phi(j) = \phi(-j) = b$, $\phi(k) = \phi(-k) = ab$.

(a) Assuming that ϕ is an epimorphism, find the elements of $K = \text{Ker}(\phi)$.

(b) Write out the distinct elements of the quotient group G/K .

(c) Give explicitly the isomorphism $\theta : G/K \rightarrow G'$ induced by ϕ .

Problem 4. (15 pts) Let $R = \{[0], [2], [4], [6], [8]\} \subseteq \mathbb{Z}_{10}$.

(a) Is R a ring with respect to the usual operations of addition and multiplication of congruence classes? Explain your answer.

(b) Does R have a unity? Explain your answer.

(c) Is R a subring of \mathbb{Z}_{10} ? Explain your answer.