

M E T U
Department of Mathematics

Basic Algebra					
MidTerm II					
Code : <i>Math 116</i>			Last Name :		
Acad. Year : <i>2013-2014</i>			Name : Student No :		
Semester : <i>Spring</i>			Department :		
Instructor : <i>G.E., T.K., M.K., A.S</i>			Signature :		
Date : <i>06.05.2014</i>			6 Questions on 4 Pages Total 60 Points		
Time : <i>17.40</i>					
Duration : <i>100 minutes</i>					
1	2	3	4	5	6

1.(10 pts.) Let $f = (1, 2, 4, 3)$ and $g = (1, 3, 5, 2)$ be permutations.

(i) Write the product fg as a product of disjoint cycles.

(ii) Write fg in as a product of transpositions.

(iii) Is fg an odd permutation or an even permutation? Give reason.

2. (10 pts.) Suppose that G is a group and K, N are normal subgroups of G .

(i) Show that $KN = \{kn \mid k \in K, n \in N\}$ is a subgroup of G .

(ii) Show that KN is a normal subgroup of G .

3. (10 pts.) If K and N are normal subgroups of G such that $|G/N| = 5$ and $|G/K| = 3$, then show that $x^{15} \in K \cap N$ for all $x \in G$.

4. (10 pts.) (i) Prove that if $G = \langle a \rangle$ is a cyclic group, then G/N is a cyclic group for any subgroup N of G .

(ii) Give an example of a non-cyclic group G and a proper normal subgroup N of G such that G/N is a cyclic group.

5. (10 pts.) (i) Let G be a group, a and g be elements in G . Prove that if a has order n , then gag^{-1} also has order n for a natural number n .

Let $f = (1, 2, 3)(4, 5)$ and $g = (2, 3, 4, 7, 8)$ be two permutations in the symmetric group.

(ii) Find gfg^{-1} .

(iii) Find the order of gfg^{-1} .

6. (10 pts.) (a) Let $G = \left\{ \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \mid a \in \mathbb{R}, a \neq 0 \right\}$ be a group of 2×2 matrices under matrix multiplication. Show that G is isomorphic to the group $\mathbb{R} - \{0\}$ (non-zero real numbers) under multiplication of numbers.

(b) Let $H = \left\{ \begin{pmatrix} b & 0 \\ 0 & b \end{pmatrix} \mid b \in \mathbb{Q}, b \neq 0 \right\}$ be the subgroup of G isomorphic to the multiplicative group of nonzero rationals.
Is H a normal subgroup of G ? (give reason)

(c) Find the cardinality of G/H . (Give reason)