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# METU MATH 116, Exam 1

Thursday, April 4, 2013, at 17:40 (100 minutes)

**Instructors:** Coskun, Karayayla, Kuzucuoglu, Solak

**Instructions:** It should be obvious to the grader how to read your solutions. Please work carefully.

**Problem 1.** (15pts)
Find the greatest common divisor $d$ of 1188 and 936. Present $d$ in the form $d = 1188x + 936y$.

**Problem 2.** (8pts) If there is one, find a solution to $17x \equiv 9 \pmod{116}$.

**Problem 3.** (12pts)
Let $x * y = x + y + k$, where $k \in \mathbb{Z}$ is fixed.

(a) Does $*$ define a binary operation on $\mathbb{Z}$? Explain your answer.

(b) Determine whether there is an identity element in $\mathbb{Z}$ for $*$.

(c) If there is an identity element, find the inverse of $x$.

**Problem 4.** (10pts) Let $G$ be the group $(\mathbb{Z}_{28}, +)$.

(a) Find all generators of $G$.

(b) List all the subgroups of $G$. For each subgroup, different from $G$, write down all of its elements.

**Problem 5.** (15pts) Let $G$ be a group with respect to multiplication and $g$ be an element of $G$.

(a) Is $C_G(g) = \{x \in G | xg = gx\}$ a subgroup of $G$? Explain your answer.

(b) Let $G = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : ad - bc \neq 0 \quad \text{where } a, b, c, d \text{ are real numbers} \right\}$ be a group with respect to matrix multiplication and let $T = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$. Find $C_G(T)$.