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METU MATH 116, Exam 1

Thursday, April 5, 2012, at 17:40 (100 minutes)

Instructions: It should be obvious to the grader how to read your solutions. Please work carefully.

Problem 1. (10pts)

Find the greatest common divisor d of 693 and 414. Present d in the form $d = 693x + 414y$ for $x, y \in \mathbb{Z}$.

Problem 2. (6pts) Let a, b and c be integers. If $\gcd(a, b) = 1$ and $c \mid a + b$, then show that $\gcd(a, c) = 1$.

Problem 3. (3+3+4 pts) Let $M_2(\mathbb{R})$ denote the set of all 2×2 real matrices. Then matrix multiplication is a binary operation on $M_2(\mathbb{R})$. Let $H = \left\{ A \in M_2(\mathbb{R}) \mid A = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} \text{ for some } a \in \mathbb{R} \right\}$.

(a) Verify that H is closed under matrix multiplication.

(b) Show that matrix multiplication is noncommutative in $M_2(\mathbb{R})$, but it is commutative in H.

(c) Show that both $M_2(\mathbb{R})$ and H contain identities for matrix multiplication, but the identities are not the same.

Problem 4. (10pts) Show that $G = \big\{(x, y) \bigm| x \in \mathbb{R}, y \in \mathbb{R} - \{0\}\big\}$ under the operation $*$ defined by $(x_1, y_1) * (x_2, y_2) = (x_1 + x_2, y_1 y_2)$ where $(x_1, y_1), (x_2, y_2) \in G$ forms a group.

Problem 5. (10pts) Solve the linear congruence $8x \equiv 66 \pmod{79}$.

Problem 6. (4+2+4+4 pts) Let $G =< g >$ be a cyclic group of order 40.

(a) Determine all **DISTINCT** subgroups of G.

(b) Find all generators of G.

(c) Find all **DISTINCT** subgroups of G of order 4.

(d) Find $H =< g^5 >$ and find all other generators of H.