M E T U
Department of Mathematics

Group	BASIC ALGEBRAIC STRUCTURES MidTerm 2	List No.

Code : *Math 116* Acad. Year : *2009-2010* Semester : *Spring* Coordinator : *M.B.,G.E.,S.P,E.S.* Date : *May.20.2010* Time : *17:40* Duration : *100 minutes*	Last Name : Name : Student No. : Department : Section : Signature : 5 QUESTIONS ON 4 PAGES TOTAL 100(+10) POINTS

1	2	3	4	5	**SHOW YOUR WORK**	

Question 1. (20 points) List the cosets of $\langle[7]\rangle$ in the multiplicative group G of $\mathbf{Z_{16}}$. Make the multiplication table for the quotient group $G/\langle[7]\rangle$.

Question 2. (20 points) Prove that every finite integral domain is a field.

Question 3. (20 points) Let $\phi : R \to S$ be a ring homomorphism between the rings R and S. Let J be an ideal of S. Show that $\phi^{-1}(J) = \{ x \in R \mid \phi(x) \in J\}$ is an ideal of R.

Question 4. (20 points) Let $\alpha = (1357)$, $\beta = (2368)$ and $\gamma = (134)(2578)(165)$ be permutations in S_8.

i) Write γ as a product of disjoint cycles.

ii) Find the order of α, β^{103}, γ.

iii) Determine whether γ is an odd or even permutation.

iv) Find a permutation $\sigma \in S_8$ such that $\sigma\alpha\sigma^{-1} = \beta$.

Question 5. Let G be a group. Define

$$Z(G) = \{ a \in G \mid ag = ga \quad \text{for all} \quad g \in G \}.$$

i)(10 pts) Show that $Z(G)$ is a normal subgroup of G.

ii)(10 pts) Prove or disprove: If $\varphi : G \to H$ is a group isomorphism between the groups G and H, then $\varphi(Z(G)) = Z(H)$.

BONUS (10 points) Let G be a group with $Z(G) = C$. If the quotient group G/C is cyclic, then G is abelian (commutative).