M E T U
Department of Mathematics

	Fundamentals of Mathematics Midterm 1		
Code : *Math 111* Acad. Year : *2017 Fall* Instructor : *G.Ercan, S.Finashin, M.Kuzucuoğlu, O.Küçüksakallı, F.Özbudak* Date : *November 14, 2017* Time : *17:40* Duration : *100 minutes*	Last Name : Name : Department : Signature :	Student No. : Section :	
	5 QUESTIONS ON 4 PAGES 100 TOTAL POINTS		

1	2	3	4	5		

1. (20pts) (a) Find all the truth values of P, Q and R such that the statement $(P \vee Q) \to (R \to \neg Q)$ is false.

P	Q	R	P∨Q	R→¬Q	(P∨Q)→(R→¬Q)	
T	T	T	T	F	F	←
T	T	F	T	T	T	
T	F	T	T	T	T	
T	F	F	T	T	T	
F	T	T	T	F	F	←
F	T	F	T	T	T	
F	F	T	F	T	T	
F	F	F	F	T	T	

∴ P : True
Q : True
R : True

OR

P : False
Q : True
R : True

(b) Determine if the statements $(\neg P) \wedge (P \to Q)$ and $\neg(Q \to P)$ are logically equivalent or not.

P	Q	P→Q	Q→P	¬P∧(P→Q)	¬(Q→P)
T	T	T	T	F	F
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	F

∴ not equivalent

2. (20pts) Determine whether the following are true or false. Explain.

(a) $\forall x \in \mathbb{R}\ \exists y \in \mathbb{R}\ \ x = y^3$. TRUE

$\forall x \in \mathbb{R}$, choose $y = \sqrt[3]{x} \in \mathbb{R}$. Such a real number y exists for any given $x \in \mathbb{R}$

(b) $\neg(\exists x \in \mathbb{R}\ \forall y \in \mathbb{R}\ \ x + y = x)$. TRUE

For any $x \in \mathbb{R}$, choos $y = 1$, then $x + 1 \neq x$ i.e $1 \neq 0$

(c) $\forall a \in \mathbb{Q}\ \forall b \in \mathbb{Q}\ \ \sqrt{ab} \in \mathbb{Q}$. FALSE

Choose $a = 1 \in \mathbb{Q}$ $b = 2 \in \mathbb{Q}$ but $\sqrt{1 \cdot 2} \notin \mathbb{Q}$

3. (20pts) Let a and b be integers. Prove that $a + b$ is even if and only if $a^3 - b^2$ is even.

$\Rightarrow$ if $a+b$ is even, then there are 2 cases for pairity of a, b

Case 1. a and b are even, then a^3 and b^2 are even and thus, $a^3 - b^2$ is even

Case 2 a and b are odd, then a^3 and b^2 are odd and thus, $a^3 - b^2$ is even

$\Leftarrow$ if $a^3 - b^2$ is even, then there are 2 cases for $a^3 - b^2$

Case 1. a^3 and b^2 are both odd; then a and b should be odd and thus, $a+b$ is even

Case 2 a^3 and b^2 are both even; then a and b should be even and thus, $a+b$ is even

Another proof of $\Leftarrow$: by contrapositive : if $a+b$ is odd, then Case 1. a is even, b is odd, then a^3 is even, b^2 is odd

Case 2. a is odd, b is even, then a^3 is odd, b^2 is even

In the both cases, $a^3 - b^2$ is odd.

4. **(20pts)** (a) Determine if the following argument is valid or not. Why ?

$$\begin{array}{l} P \to (Q \vee R) \\ \underline{\neg P \vee Q} \\ Q \end{array}$$

Not valid.

Put P = False, Q = False Then

- $P \to (Q \vee R) = \text{True}$ independent of R
- $\neg P \vee Q = \text{True}$

But Q = False

(b) Give a derivation of the following argument.

$$\begin{array}{ll} ① & E \to F \\ ② & \neg G \to \neg F \\ ③ & H \to I \\ ④ & \underline{E \vee H} \\ & G \vee I \end{array}$$

⑤ $F \vee I$ by 4, 1, 3

⑥ $F \to G$ by 2

⑦ $G \vee I$ by 5, 6

5. (20pts) For sets A and B the *symmetric difference* is defined as $A \Delta B = (A - B) \cup (B - A)$. Prove the following

$$X \cap (Y \Delta Z) = (X \cap Y) \Delta (X \cap Z).$$

$\subseteq$ Take any $x \in X \cap (Y \Delta Z)$, then $x \in X$ and $x \in Y \Delta Z = (Y - Z) \cup (Z - Y)$, thus, $x \in X$ and either $x \in Y - Z$ or $x \in Z - Y$

Case 1. $x \in X$ and $x \in Y - Z$, then $x \in X$, $x \in Y$ and $x \notin Z$, so, $x \in X \cap Y$, but $x \notin X \cap Z$, thus $x \in (X \cap Y) - (X \cap Z)$.

Case 2 $x \in X$ and $x \in Z - Y$, then $x \in X$, $x \in Z$, but $x \notin Y$, so, $x \in X \cap Z$, but $x \notin X \cap Y$, and thus, $x \in (X \cap Z) - (X \cap Y)$

In the both cases, $x \in (X \cap Y) \Delta (X \cap Z) = [(X \cap Y) - (X \cap Z)] \cup [(X \cap Z) - (X \cap Y)]$

$\supseteq$ Take any $x \in (X \cap Y) \Delta (X \cap Z) = [(X \cap Y) - (X \cap Z)] \cup [(X \cap Z) - (X \cap Y)]$

Case 1 $x \in (X \cap Y) - (X \cap Z)$, then $x \in X \cap Y$ and $x \notin X \cap Z$, then $x \in X$, $x \in Y$, but $x \notin X \cap Z$ and thus, $x \notin Z$

Then $x \in Y - Z \subseteq (Y - Z) \cup (Z - Y) = Y \Delta Z$

Case 2 $x \in (X \cap Z) - (X \cap Y)$, then $x \in X \cap Z$ and $x \notin X \cap Y$, then $x \in X$, $x \in Z$, but $x \notin Y$. Thus $x \in Z - Y \subseteq Y \Delta Z$

In the both cases, $x \in X \cap (Y \Delta Z)$.

Another Solution: $X \cap (Y \Delta Z) = X \cap ((Y - Z) \cup (Z - Y)) = [X \cap (Y - Z)] \cup [X \cap (Z - Y)]$

distributivity

$X \cap (Y - Z) = (X \cap Y) - (X \cap Z)$
$X \cap (Z - Y) = (X \cap Z) - (X \cap Y)$

implies $X \cap (Y \Delta Z) = [(X \cap Y) - (X \cap Z)] \cup [(X \cap Z) - (X \cap Y)]$

$= (X \cap Y) \Delta (X \cap Z)$