Department of Mathematics, METU, MATH 111 - FUNDAMENTALS OF MATHEMATICS

FIRST MIDTERM

STUDENT NUMBER	NAME, FAMILY NAME	GRADE

13th November 2015. Duration : 100 minutes. Four questions : 20 + 5 , 25 , 9 + (4 + 4 + 4 + 4) , 25

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1. Construct the truth table of the following statement :

$$\Big((\neg P) \longrightarrow Q\Big) \longrightarrow \Big(P \vee R\Big)$$

Is this statement a tautology ? A contradiction ?

P	Q	R	$(($¬$P)$	$\longrightarrow$	$Q)$	$\longrightarrow$	$(P$	$\vee$	$R)$
T	T	T	F	T	T	T	T	T	T
T	T	F	F	T	T	T	T	T	F
T	F	T	F	T	F	T	T	T	T
T	F	F	F	T	F	T	T	T	F
F	T	T	T	T	T	T	F	T	T
F	T	F	T	T	T	F	F	F	F
F	F	T	T	F	F	T	F	T	T
F	F	F	T	F	F	T	F	F	F

This is neither a tautology nor a contradiction...

2. Using the notation

$$
\begin{aligned}
A &: \text{"Ayşe goes to the dance"} \\
B &: \text{"Betül is angry"} \\
E &: \text{"Eşref plays cards all night"} \\
R &: \text{"Rauf is angry"} \\
Z &: \text{"Zeynep is notified"}
\end{aligned}
$$

show that the following argument is valid by writing a derivation :

"If Ayşe goes to the dance, then Betül will be angry. If Eşref plays cards all night, then Rauf will be angry. If Betül or Rauf is angry, then Zeynep will be notified. Zeynep was not notified. It follows that Ayşe did not go to the dance and Eşref did not play cards all night."

1.	$A \to B$	The premises ...
2.	$E \to R$	
3.	$(B \vee R) \to Z$	
4.	$\neg Z$	
5.	$\neg Z \to \neg(B \vee R)$	From 3, by "contraposition".
6.	$\neg(B \vee R)$	From 4 & 5 by MP ($P \to Q$ & $P \Rightarrow Q$).
7.	$\neg B \wedge \neg R$	From 6 by "De Morgan" rules.
8.	$\neg B$	From 7 by "simplification".
9.	$\neg R$	
10.	$\neg B \to \neg A$	From 1 by "contraposition".
11.	$\neg R \to \neg E$	" 2 " " .
12.	$\neg A$	From 8 & 10 by MP.
13.	$\neg E$	" 9 " 11 " "
14.	$\neg A \wedge \neg E$	Conjunction of 12 & 13.

3. (A) Find the negation of the statement

$$(\forall x \in \mathbb{Z})(\exists y \in \mathbb{Z})\Big\{\Big(x+y=2\Big) \vee \Big(x-2y \neq 3\Big)\Big\}.$$

(Your result should not contain the negation sign "$\neg$" !)

(B) Let $P(x,y)$ stand for $x > y^2$ where x, y take integer values. Decide if the following statements are true or false :

(1) $(\forall x \in \mathbb{Z})(\exists y \in \mathbb{Z})P(x,y)$, (2) $(\forall y \in \mathbb{Z})(\exists x \in \mathbb{Z})P(x,y)$,

(3) $(\exists y \in \mathbb{Z})(\forall x \in \mathbb{Z})P(x,y)$, (4) $(\exists x \in \mathbb{Z})(\forall y \in \mathbb{Z})P(x,y)$.

Explain your answer briefly in each case.

(A) $\neg(\forall x \in \mathbb{Z})(\exists y \in \mathbb{Z})\{(\dots)\vee(\dots)\}$

$\iff (\exists x \in \mathbb{Z})\,\neg(\exists y \in \mathbb{Z})\{(\ )\vee(\ )\}$ Negation of the universal quantifier

$\iff (\exists x \in \mathbb{Z})(\forall y \in \mathbb{Z})\,\neg\{(\ )\vee(\ )\}$ Negation of the existential quantifier.

$\iff (\exists x \in \mathbb{Z})(\forall y \in \mathbb{Z})\{(x+y\neq 2)\wedge(x-2y=3)\}$ The "De Morgan" rule.

(B) 1) $(\forall x \in \mathbb{Z})(\exists y \in \mathbb{Z})(x>y^2)$ is false, since ~~x~~ $x=-5$ admits no y with $-5>y^2\geq 0$.

2) $(\forall y \in \mathbb{Z})(\exists x \in \mathbb{Z})(x>y^2)$ is true since for given y $x \geq y^2$ holds for all sufficiently large $x\in\mathbb{Z}$.

3) $(\exists y \in \mathbb{Z})(\forall x \in \mathbb{Z})(x>y^2)$ is false since whatever y may be $x>y^2$ is false for negative x.

4) $(\exists x \in \mathbb{Z})(\forall y \in \mathbb{Z})(x>y^2)$ is false since for any value of x, $y^2 \geq x$ for any y with $y\geq\sqrt{|x|}$.

4. Prove that $B - C \subseteq A$ if and only if $B - A \subseteq C$ for any sets A, B, C.

Suppose that $B - C \subseteq A$: (*)

For any $x \in B - A$, we have $(x \in B) \wedge (x \notin A)$.

As $B - C \subseteq A$, it is clear that $[(x \in B) \wedge (x \notin C)] \longrightarrow x \in A$.

Taking the contrapositive gives

$$(x \notin A) \longrightarrow [(x \notin B) \vee (x \in C)]$$

From (*) we have – by simplification – $(x \notin A)$ which gives us

$$x \notin B \vee x \in C.$$

However, ~~the~~ $x \in B - A$, hence $x \in B$ or equivalently $\neg (x \notin B)$. It follows that $x \in C$.

This being true for arbitrary x, it is seen that

$$B - A \subseteq C.$$

The converse can be established by interchanging the roles of A and C!