2010 MT1.

| | |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| Σ | |

Name and Surname:
Student number:

# METU MATH 111, EXAM 1

Tuesday, November 9, 2010, at 17:40

**Instructors:** Berkman, Küçüksakallı, Pamuk, Pierce

**Instructions:** There are 7 numbered problems on 4 pages. It should be obvious to the grader how to read your solutions. Please work carefully.

**Problem 1.** Complete the following 'full' truth-table. (You should write the possible values of the variables in the standard order. The symbol & has the same meaning as $\wedge$.)

| $(P$ | $\vee$ | $\neg$ | $Q)$ | $\Rightarrow$ | $(R$ | & | $Q)$ |
|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |

**Problem 2.** Find a formula $F$ with the following truth-table. Explain briefly what you did.

| $P$ | $Q$ | $R$ | $F$ |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 0 |

Disjunctive Normal Form:

$(\neg P \wedge \neg Q \wedge \neg R) \vee (P \wedge \neg Q \wedge \neg R) \vee (\neg P \wedge Q \wedge R)$

**Problem 3.** Write down the *negation* of the following sentence, without using the negating connective $\neg$ (you may use the sign of inequality $\neq$).

$$\forall x\, \forall y\, (x^2 = y^2 \implies x = y \vee x = -y).$$

$$\exists x\, \exists y\, (x^2 = y^2 \wedge x \neq y \wedge x \neq -y)$$

**Problem 4.** Establish the following logical entailment by the method of your choice. (You may for example write a formal proof in the 'System of Detachment', or do something else, as long as it is clear to the reader what you are doing. The symbol & has the same meaning as $\wedge$.)

$$P,\ \neg P \vee Q,\ Q \Rightarrow R,\ \neg(R \,\&\, \neg S) \models S.$$

| | Steps | Reasons |
|---|---|---|
| 1) | $\neg P \vee Q$ | Premise |
| 2) | $P \Rightarrow Q$ | Defn of $\Rightarrow$ |
| 3) | $Q \Rightarrow R$ | Premise |
| 4) | $P \Rightarrow R$ | Hyp. Syl. |
| 5) | $P$ | Premise |
| 6) | $R$ | Detachment |
| 7) | $\neg(R \wedge \neg S)$ | Premise |
| 8) | $\neg R \vee S$ | De Morgan |
| 9) | $R \Rightarrow S$ | Defn of $\Rightarrow$ |
| 10) | $S$ | Detachment |

**Problem 5.** Assume that the universe consists of the integers (that is, all variables stand for integers); let $E(x)$ stand for the property of $x$ being even; and let $P(x)$ stand for $x$ being prime. Using the quantifiers $\forall$ and $\exists$ as needed, write down symbolically the following statement:

Every even integer that is greater than 4 is the sum of two primes.

$$\forall x \left[ E(x) \land x > 4 \Rightarrow \exists a \exists b \left( P(a) \land P(b) \land x = a+b \right) \right]$$

**Problem 6.** Assuming $A$ and $B$ are arbitrary sets, prove

$$A \not\subseteq B \iff A \cap B^c \neq \varnothing.$$

$\Rightarrow$) Suppose that $A$ is not a subset of $B$. Then there exists an element $x \in A$ which is not in $B$. In other words $x \in A$ and $x \in B^c$. This means that $x \in A \cap B^c$ and therefore this intersection is not empty.

$\Leftarrow$) Conversely suppose that $A \cap B^c$ is not empty. Hence we can pick an element $x$ in this intersection. Such an element satisfies $x \in A$ and $x \in B^c$. This follows that $A$ is not a subset of $B$. This finishes the proof.

**Problem 7.** Explain what is wrong with the following 'proof' of the claim that, for any sets $A$, $B$, $C$, and $D$,

$$(A \cup B) \cap (C \cup D) \subseteq (A \cap C) \cup (B \cap D). \qquad (*)$$

(Parts of the proof are numbered for ease of reference.)

(a) Suppose $x \in (A \cup B) \cap (C \cup D)$;
(b) we shall show $x \in (A \cap C) \cup (B \cap D)$.
(c) We have $x \in A \cup B$
(d) and $x \in C \cup D$.
(e) Therefore $x \in A$ or $x \in B$;
(f) also, $x \in C$ or $x \in D$.
(g) If $x \in A$ and $x \in C$, then $x \in A \cap C$.
(h) If $x \in B$ and $x \in D$, then $x \in B \cap D$.
(i) Since $A \cap C \subseteq (A \cap C) \cup (B \cap D)$
(j) and $B \cap D \subseteq (A \cap C) \cup (B \cap D)$,
(k) it follows that $x \in (A \cap C) \cup (B \cap D)$.
(l) Therefore $(*)$ holds.

The line (k) does not follow from the previous lines.