

GROUP THEORY in Ankara

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II

September 14-15
2026

in honour of
Mahmut
Kuzucuoğlu's
retirement



ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY



TÜRK
MATEMATİK
DERNEĞİ

Group Theory in Ankara – II
14 - 15 September 2026
Middle East Technical University

Welcome to the conference “Group Theory in Ankara – II”, organized in honour of Prof. Mahmut Kuzucuoğlu’s retirement. Thank you for joining us in celebrating Prof. Kuzucuoğlu’s remarkable mathematical career and his contributions to group theory.

We are delighted to see such a large number of participants. This is not only a reflection of the continuing interest in the theory of groups, but also a testament to the respect and affection that Prof. Kuzucuoğlu has earned throughout his career. Each of us has our own reason for attending this conference. Some of us are Kuzucuoğlu’s students, some are his colleagues and collaborators, and others are his friends. In all cases, we are here to celebrate his scientific contributions, as well as the friendship and collegiality he has shared with so many of us.

When we began organizing this conference, our aim was to bring together researchers at different stages of their careers who share an interest in group theory. We are grateful to all the participants and speakers for contributing to this gathering. We would also like to extend our special thanks to those who have traveled long distances to join us. Your presence and support make this celebration all the more meaningful.

We would like to thank Middle East Technical University and the Turkish Mathematical Society for their generous support of the conference.

We hope you enjoy the scientific program, the discussions, and the opportunity to meet and reconnect with colleagues and friends. Most importantly, we hope you have a memorable time celebrating Prof. Kuzucuoğlu and his remarkable career with us.

14 September 2026
Organizing Committee

Group Theory in Ankara – II
in honor of Mahmut Kuzucuoğlu's retirement

Program

September, 2026
Middle East Technical University

Monday, September 14

Time	Event
8:30–9:00	Registration
9:00–9:20	Opening Ceremony
9:20–10:10	Pavel Shumyatsky
10:10–10:20	Questions and Comments
10:20–10:40	Coffee Break
10:40–11:30	Mercede Maj
11:30–11:40	Questions and Comments
11:40–12:00	Coffee Break
12:00–12:10	Conference Photograph
12:20–13:50	Lunch
14:00–14:50	Mattia Brescia
14:50–15:00	Questions and Comments
15:00–15:20	Coffee Break
15:20–16:10	Bogdana Oliynyk
16:10–16:20	Questions and Comments
16:20–16:40	Coffee Break
16:40–16:50	Obituary Speech: Vladimir Levchuk
16:50–17:40	Kıvanç Ersoy
17:40–17:50	Questions and Comments
18:00–19:00	Transportation
19:00–21:00	Conference Dinner

Group Theory in Ankara – II
in honor of Mahmut Kuzucuoğlu's retirement

Program

September, 2026
Middle East Technical University

Tuesday, September 15

Time	Event
9:20–10:10	Patrizia Longobardi
10:10–10:20	Questions and Comments
10:20–10:40	Coffee Break
10:40–11:30	Cristina Acciarri
11:30–11:40	Questions and Comments
11:40–12:00	Coffee Break
12:10–13:50	Lunch
14:00–14:10	Obituary Speech: Karim Ahmadidelir
14:10–15:00	Mohammad Shahryari
15:00–15:10	Questions and Comments
15:10–15:30	Coffee Break
15:30–15:40	Obituary Speech: Francesco de Giovanni
15:40–16:30	Marco Trombetti
16:30–16:40	Questions and Comments
16:40–17:00	Coffee Break
17:00–17:50	Hayrullah Ayık
17:50–18:00	Questions and Comments
18:00–18:20	Closing Ceremony

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– <i>On the Ranks and Generating Sets of Certain Transformation Semi-groups</i>	
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On Profinite Groups With Restricted Centralizers of Powers

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A group G is said to have restricted centralizers if for every $x \in G$ the centralizer $C_G(x)$ either is finite or has finite index in G . Conditions on centralizers often impose strong structural constraints on profinite groups. A notable example is a theorem of Shalev asserting that profinite groups with restricted centralizers are virtually abelian.

In this talk, I will discuss the following recent result obtained jointly with Pavel Shumyatsky.

Theorem. *Let n be a positive integer and G a profinite group in which the centralizer of x^n is either finite or open for every $x \in G$. Then G has an open normal subgroup T such that $G/Z(T)$ has finite exponent.*

I will also explain how this problem fits into a broader framework involving probabilistic group theory.

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* Joint work with Pavel Shumyatsky (University of Brasilia)

On the Ranks and Generating Sets of Certain Transformation Semigroups

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For $n \in \mathbb{N}$, let $\mathcal{PT}_n$, $\mathcal{T}_n$ and $\mathcal{I}_n$ be the partial, full and partial injective transformation semigroups on the finite chain $X_n = \{1 < \cdots < n\}$, respectively. We denote

- the subsemigroup of $\mathcal{PT}_n$ consisting of all order-preserving and order-decreasing partial transformations on X_n by $\mathcal{PC}_n$;
- the subsemigroup of $\mathcal{PT}_n$ consisting of all monotone and order-decreasing partial transformations on X_n by $\mathcal{PMD}_n$;
- the subsemigroup of $\mathcal{T}_n$ consisting of all oriented and order-decreasing full transformations on X_n by $\mathcal{ORD}_n$;
- the subsemigroup of $\mathcal{PT}_n$ consisting of all oriented and order-decreasing partial transformations on X_n by $\mathcal{PORD}_n$; and
- the subsemigroup of $\mathcal{I}_n$ consisting of all oriented and order-decreasing injective partial transformations on X_n by $\mathcal{IORD}_n$.

Moreover, for $2 \leq r \leq n$, let

$$\mathcal{U}(n, r) = \{\alpha \in \mathcal{U}_n : |\text{im}(\alpha)| \leq r\},$$

where $\mathcal{U}_n \in \{\mathcal{PMD}_n, \mathcal{ORD}_n, \mathcal{PORD}_n, \mathcal{IORD}_n\}$. In this talk, we present the results we obtained regarding some minimal generating sets, the ranks and the maximal subsemigroups of $\mathcal{U}(n, r)$ for $1 \leq r \leq n$, explaining how we roughly determined them.

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κ -Existentially Closed Groups and Centralizers

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Let κ be an infinite cardinal. The class of κ -existentially closed groups show an interesting interplay between rigidity and abundance in their structure. For uncountable κ , groups of cardinality κ are unique up to isomorphism; however, as soon as we allow the cardinality of the group to be larger, this uniqueness fails in the strongest possible way.

In this talk I will discuss this phenomenon and, in particular, the role played by centralizers in the structure of κ -existentially closed groups. If G has cardinality κ and $A \leq G$ is generated by fewer than κ elements, we characterize exactly when $C_G(A)/Z(A)$ is isomorphic to G . As a further application, centralizers provide inside G a family of subgroups isomorphic to G itself whose inclusion order has the order type of the rational numbers.

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* Joint work with Kivanç Ersoy and Mahmut Kuzucuoğlu.

Minimal and ω -Non-Sofic Groups: Structure and Recent Examples

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In our earlier paper [1], written when the existence of non-sofic groups was still an open problem, we introduced two notions for studying the possible subgroup structure of such groups. A non-sofic group is called *minimal non-sofic* if all its proper subgroups are sofic, and *ω -non-sofic* if it admits a strictly descending infinite chain of non-sofic subgroups.

The situation has since changed: the first non-sofic group was constructed by OpenAI [5], and further examples were obtained by Fournier–Facio [3] and by Kun and Thom [4].

We first discuss some results from our earlier paper. In particular, if G is a minimal non-sofic group and M is a finitely generated residually finite maximal normal subgroup of G , then M is central and G is a perfect central extension of a finitely generated non-amenable simple group. We also proved that every locally graded non-sofic group is ω -non-sofic.

In [2], we study some of the recently constructed non-sofic groups. We prove that the unit group of the binary Leavitt algebra, shown to be non-sofic by OpenAI, and the non-sofic generalized wreath products constructed by Kun and Thom admit proper self-embeddings. Hence these groups are ω -non-sofic. For the explicit Kun–Thom family, we describe a strictly descending chain of finite-index normal subgroups, all of which are locally graded and non-sofic. In particular, not every locally graded group is sofic.

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On a New Problem Related to Centralizers of Elements in Groups

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Let G be a group, let x be an element of G , and denote by $C_G(x)$ the centralizer of x in G , that is, $C_G(x) = \{a \in G \mid ax = xa\}$. A nontrivial element $x \in G$ is called *deficient* if $\langle x \rangle < C_G(x)$, and *non deficient* if $\langle x \rangle = C_G(x)$. A conjugacy class x^G is called *deficient* if x is deficient, and *non deficient* if x is non-deficient. If j is a non-negative integer, we say that a group G belongs to $D(j)$, or that G is a $D(j)$ group, if it has exactly j nontrivial deficient conjugacy classes.

We study the exact value of j for which $G \in D(j)$, for groups G belonging to some classes of groups, in particular for symmetric groups and dihedral groups ([1]). We then present some recent results on groups in $D(0)$ and in $D(1)$ ([3]). We also consider groups G in the class $D(j)$ containing an element of order p^{j+1} ([5]). Finally, we report some known results on finite and locally finite groups in which every element has order a power of a prime ([2]), together to some new results about some classes of infinite groups with this property ([2]).

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* Joint work with B. Cohen, M. Herzog, M. Maj

Some Problems Related to Centralizers of Elements in Groups

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Let G be a group, let x be an element of G , and denote by $C_G(x)$ the centralizer of x in G , that is,

$$C_G(x) = \{a \in G \mid ax = xa\}.$$

We investigate several problems concerning centralizers of elements in groups. In the first part of the talk, we present some results of Mahmut Kuzucuoğlu on centralizers of elements in locally finite simple groups, mainly those contained in the papers [4], [5], and [6].

In the second part, we present recent results on groups G such that, for every nontrivial element x in G , $\langle x \rangle < C_G(x)$. We call such groups D -groups (see [1], [2], and [3]). We study, in particular, simple groups, and show that every finite simple group is a D -group, whereas an infinite locally finite group is never is a D -group. We then investigate nilpotent groups that are D -groups. Finally, we present some interesting examples of D -groups. In particular, we show that the Heisenberg groups over $\mathbb{Z}n$, for $n > 2$, are D -groups.

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* Joint work with B. Cohen, M. Herzog, P. Longobardi

Cayley Graphs of Alexander and Generalized Alexander Quandles

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A quandle was introduced as an algebraic system whose axioms arise from the Reidemeister moves for classical knot diagrams. In particular, classical knots are completely classified by their fundamental quandles [2]. Quandles also arise naturally in Hopf algebras, set-theoretic solutions of the Yang–Baxter equation, and the theory of symmetric spaces.

There is a natural connection between groups and quandles. Several important classes of quandles, including conjugation quandles, Alexander quandles, and generalized Alexander quandles, are defined using groups and their automorphisms. Conversely, the symmetries of a quandle can be studied through its automorphism group.

In this talk, we investigate the Cayley graphs of quandles arising from groups. Our aim is to understand how algebraic properties of the underlying group are reflected in the graph-theoretic structure of these Cayley graphs. In particular, for an Alexander quandle $A_t(G)$ over a finite abelian group G , we prove that the connected components of the Cayley graph correspond to the cosets of the subgroup $\text{im}(\text{id} - t)$. For a generalized Alexander quandle $A_\phi(G)$ over a finite group G , we show that its Cayley graph is regular and that the in-degree and out-degree of every vertex are equal to the index in G of the subgroup of fixed points of the automorphism ϕ . Finally, when the defining automorphism is inner, we prove that the Cayley graph is strongly connected if and only if G is perfect and the element defining the inner automorphism normally generates G [1].

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* Joint work with David Dolžan

Residually A -Free Groups and Conjugately Separable $\mathfrak{X}$ -Subgroups

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A subgroup H of a given group G is called *malnormal* if for every element $x \in G \setminus H$, we have $H \cap H^x = \{1\}$. A group G is called *conjugate separable abelian* (CSA) if all maximal abelian subgroups of G are malnormal. The class of CSA-groups is broad and plays a significant role in the study of residually free groups, universal theory of non-abelian free groups, limit groups, exponential groups and equational domains in algebraic geometry over groups (see [2], [3], [12], and [13]). Another class of groups with very close connections to CSA-groups is the class of CT-groups (*commutative transitive groups*). A group is CT if commutativity is a transitive relation on the set of its non-identity elements. Despite this simple definition, the class of CT groups is also central to the study of residually free groups. Every CSA-group is CT but the converse is not true. B. Baumslag proved that in the presence of residual freeness, both properties are equivalent.

In recent decades, considerable effort has been devoted to the study of these classes and their generalizations. A generalization of CT-groups is introduced in [4] to extend the above mentioned theorem of B. Baumslag. Many interesting relations between CSA- and CT-groups are presented in [6], along with an extensive review of the existing literature.

I proposed an idea in [18] for generalizing the notions of CSA- and CT-groups: Suppose that $\mathfrak{X}$ is a class of groups. A group G is an $\mathfrak{X}$ T-group if and only if, for any two $\mathfrak{X}$ -subgroups A and B of G , the assumption $A \cap B \neq \{1\}$ implies that $\langle A, B \rangle$ is also an $\mathfrak{X}$ -subgroup of G . Similarly, a group G is a CS $\mathfrak{X}$ -group if all of its maximal $\mathfrak{X}$ -subgroups are malnormal. In [18], I examined the special case where $\mathfrak{X} = \mathfrak{N}_k$, the variety of nilpotent groups of nilpotency class not exceeding k , which contains CSA- and CT-groups as particular instances. Hence, [18] addresses the study of general relations among the classes of CS $\mathfrak{X}$ - and $\mathfrak{X}$ T-groups, their characterizations, constructions, and universal axiomatization, as well as their connections to residual A -free groups, in the case when $\mathfrak{X}$ is the variety of nilpotent groups of class at most k . Furthermore, many previous results are shown to remain valid for CS $\mathfrak{X}$ and $\mathfrak{X}$ T-groups when $\mathfrak{X}$ is the variety $\mathfrak{N}_k$. As an application, the ideas of [18] are used in [14] to introduce a large class of groups which are *equational domain* in the sense of algebraic geometry over groups (see [2] for definitions).

It is well-known that the classes of CT- and CSA-groups are closely related to the classes of residually free and fully residually free groups, and therefore play an important role in the study of the universal theory of non-abelian free groups (see [1], [3], and [16]). In [1] it is proved that in the presence of the property of being residually free, every CT-group belongs to the class CSA, a

fact that is generalized to the case $\mathfrak{X} = \mathfrak{N}_k$ in [18]. We shall extend all this theory to a wider class of varieties.

Let A be a group. An A -free group is a free product of a set of copies of A . Clearly, if A is an infinite cyclic group, then A -free groups are the ordinary free groups.

The following two facts can be verified (see [18]):

1. If $|A| \geq 3$, then $A * A * A$ embeds in $A * A$.
2. If $|A| \geq 3$, then the free group $\mathbb{F}_2$ embeds in $A * A$.

In what follows we shall always assume that A has no involution. A group G is residually A -free if, for every non-identity element g in G , there is a homomorphism from G to an A -free group F such that g maps to a non-identity element of F . We say that G is fully residually A -free if, for any finite collection of distinct non-trivial elements $g_1, \dots, g_n$ of G , there is a homomorphism from G to an A -free group F that maps each of $g_1, \dots, g_n$ to a non-identity element of F . Observe that every (fully) residually free group is (fully) residually A -free whenever A is a group having no involutions. Here is our main results:

Theorem A. Suppose $\mathfrak{X}$ is a finitely based variety such that the relatively free element $F_2(\mathfrak{X})$ is finitely presented. Let $A \in \mathfrak{X}$ be a non-trivial finitely generated equationally Noetherian group with no involutions and assume that $\text{CS}\mathfrak{X} \subseteq \mathfrak{X}\text{T}$. Then, every fully residually A -free group belongs to $\text{CS}\mathfrak{X}$.

Theorem B. Let $\mathfrak{X}$ be a variety which contains all abelian groups and suppose that $\text{CS}\mathfrak{X} \subseteq \mathfrak{X}\text{T}$. Let $A \in \mathfrak{X}$ be a group without any involution. Then, every residually A -free $\mathfrak{X}\text{T}$ -group belongs to $\text{CS}\mathfrak{X}$.

As a special case of the theorem B, we have the following corollary:

Corollary. Let $\mathfrak{X}$ be a variety which contains all abelian groups and suppose that $\text{CS}\mathfrak{X} \subseteq \mathfrak{X}\text{T}$. Then, every residually free $\mathfrak{X}\text{T}$ -group belongs to $\text{CS}\mathfrak{X}$.

We have also the the following result.

Theorem C. Let $\mathfrak{X}$ be a variety containing all abelian groups and assume that $\text{CS}\mathfrak{X} \subseteq \mathfrak{X}\text{T}$. Let $A \in \mathfrak{X}$ be a group with no involutions, and let $G \in \mathfrak{X}\text{T} \setminus \mathfrak{X}$ be residually A -free. Then, G is fully residually A -free.

If we consider the special case where A is an infinite cyclic group, then we obtain the following result.

Corollary. Let $\mathfrak{X}$ be a variety containing all abelian groups and assume that $\text{CS}\mathfrak{X} \subseteq \mathfrak{X}\text{T}$. Let $G \in \mathfrak{X}\text{T} \setminus \mathfrak{X}$ be residually free. Then, G is fully residually free.

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Probabilistically Nilpotent Finite Groups (And Some Other Topics)

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There are many ways to characterize finite nilpotent groups. For example, a finite group G is nilpotent if and only if any two Sylow subgroups of coprime orders commute. In this talk we will discuss finite groups in which the above feature holds with high probability and we will see how this affects the structure of the group. This will be the second part of the talk.

The first part will be about my earlier work with Mahmut Kuzucuoglu.

On the Behaviour of Closed Subgroups in Linear Groups

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It is well known that any linear group, endowed with the Zariski topology, satisfies the minimal condition on closed subgroups and the maximal condition on closed connected subgroups. In this talk, I will describe the behaviour of linear groups satisfying the maximal condition on arbitrary closed (normal) subgroups, with a special focus on Zariski-simple linear groups, that is, linear groups with no proper non-trivial closed normal subgroups.

* Joint work with F. de Giovanni† and B.A.F. Wehrfritz