Locally finite groups with bounded centralizer chains and Borovik's conjecture

Alexander Buturlakin

The *c*-dimension of a group is the maximal length of a nested centralizer chain. We are concerned with locally finite groups of finite *c*-dimension. More precisely, we study the following conjecture by A. Borovik.

Conjecture. Let G be a locally finite group of finite c-dimension k. Let S be the full inverse image of $F^*(G/F(G))$ in G. Then

(1) the number of nonabelian simple composition factors of G is finite and k-bounded;

(2) G/S has an abelian subgroup of finite k-bounded index.

Here F(G) denotes the Hirsch–Plotkin radical of G, which is the largest normal locally nilpotent subgroup of G, and $F^*(G)$ denotes the generalized Fitting subgroup of G, which is the subgroup generated by F(G) and all the quasisimple subnormal subgroups.

We prove that the first part of the conjecture is true, while the second one is false. Also we establish some weaker version of the second part for finite groups G.