PRONORMALITY AND THE FRATTINI ARGUMENT FOR HALL SUBGROUPS

DANILA REVIN

The following simple statement is frequently used in the finite group theory.

The Frattini argument. *Let A be a normal subgroup of a finite group G and let S be a Sylow p-subgroup of A for a prime p. Then $G = AN_G(S)$.*

A subgroup H of a group G is said to be *pronormal* if H and H^g are conjugate in $\langle H, H^g \rangle$ for every $g \in G$. If H is a pronormal subgroup of a group G and is contained in a normal subgroup A of G then the Frattini Argument holds for H, i. e. $G = AN_G(H)$. The following strengthening of the Frattini argument holds: *If A is a normal subgroup of G and P is a Sylow p-subgroup of A then P is pronormal in G.*

There is a natural generalization of the concept of a Sylow p-subgroup. Take a set π of primes. A subgroup H of a group G is called a *Hall π-subgroup* if every prime divisor of $|H|$ belongs to π and $|G : H|$ is not divisible by the elements of π. Every Sylow p-subgroup is a Hall π-subgroup for $\pi = \{p\}$.

If a group G is solvable then the Hall theorem implies that the Hall π-subgroups of normal subgroups are pronormal in G. But in the case where G is non-solvable, even the Frattini argument fails for Hall π-subgroups in general. In the talk, we discuss a weaker analog of the Frattini argument and the problem of existence of pronormal Hall π-subgroups in a given group.

According to P. Hall, we write $G \in \mathcal{E}_\pi$ if G contains a Hall π-subgroup. We prove the following statements:

Theorem. *Every normal subgroup A of a group $G \in \mathcal{E}_\pi$ contains a Hall π-subgroup which is pronormal in G.*

Corollary 1. *If G possesses a Hall π-subgroup then G possesses a pronormal Hall π-subgroup.*

Corollary 2 . *Every normal subgroup A of a group $G \in \mathcal{E}_\pi$ contains a Hall π-subgroup H such that $G = AN_G(H)$.*

Sobolev Institute of Mathematics, Novosibirsk (Russia)
E-mail address: revin@math.nsc.ru