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METU MATH 116, Final Exam

Friday, June 7, 2013, at 9:30 (120 minutes)

Instructors: Coşkun, Karayayla, Kuzucuoğlu, Solak

Instructions: It should be obvious to the grader how to read your solutions. Please work carefully.

Problem 1. (12pts)
Prove that if H is a subgroup of G with $|G : H| = 2$, then H is a normal subgroup of G.

Problem 2. (12pts)

(a) Show that the polynomial, $f(x) = 2x^3 + x^2 + x + 1$ is irreducible in $\mathbf{Q}[x]$.

(b) Why is $f(x)$ reducible in $\mathbf{R}[x]$?

Problem 3. (12pts)
Find a polynomial $f(x)$ of least degree with the given property:
$f(x)$ is over real numbers and $-3i$ and $1 - i$ are zeros of it.

Problem 4. (12pts)

(a) Find gcd of $f(x) = x^3 - 2x^2 + x - 2$ and $g(x) = x^2 - x - 2$ in $\mathbf{R}[x]$.

(b) Express your gcd as a linear combination of the given polynomials $f(x)$ and $g(x)$.

Problem 5. (12pts)

(a) Prove that if $\alpha : R \to S$ is a surjective (onto) ring homomorphism, then for every ideal I in R the image $\alpha(I)$ is an ideal in S.

(b) Give an example to show that if α is not surjective, then $f(I)$ need not be an ideal. **Show your work clearly.**