**MATH 353 Complex Calculus**

**Credit:** (4-0) 4

**Catalog description:** Algebra of complex numbers. Polar representation. Analyticity. Cauchy-Riemann equations. Power series. Elementary functions. Mapping by elementary functions. Linear fractional transformations. Line integral. Cauchy-Theorem. Cauchy integral formula. Taylor`s Series. Laurent series. Residues. Residue theorem. Improper integrals.

**Course Objectives:** By the end of this course, a student will:

* Understand in detail the complex number system and the complex plane,
* Use functions of a complex variable and explore their properties
* Understand the concepts of the derivative of a complex function and analyticity of a function,
* Make calculations with elementary functions of a complex variable
* Evaluate line integrals and prove results using the Cauchy-Goursat theorem and the Cauchy integral formulas,
* Master the concepts of complex sequences and infinite series, Laurent series, residues and the residue theorem and apply them to various problems
* Understand the basics of conformal mappings and Möbius transformations

**Course Website:** <https://metuclass.metu.edu.tr/>

**Textbook:** “Complex Variables and Applications”, Brown, J. W., Churchill, R. V., 8th ed.

**Reference Material:** “Complex Analysis”, Gamelin, T. W.

“Basic Complex Analysis”, Marsden, J. E. , Hoffman, M. J.

“Principles of Mathematical Analysis”, Rudin, W.

**Exams and Grading:**

Midterm 1 : 30 % - Nov 13, 2017

Midterm 2 : 30 % - Dec 18, 2017

Final : 40 % (During Finals’ week, date TBA)

Bonus : 4 points for 90% or higher attendance. In addition, some exams may contain bonus problems

**Attendance:** A minimum of 70% attendance is required. A student attending less than 70% of the lectures will receive an NA grade.

**Suggested Problems**:A list of suggested problems will be announced on METUClass. Students are encouraged to attempt to solve all of these problems in a timely manner and discuss their solutions with the course instructors during office hours. At least 25% of the exam problems will be chosen among these problems.

**NA Policy:** A student who misses all exams or who fails to meet the attendance requirement will receive a grade of NA for the course.

**Make-up Policy:** In order to be eligible to enter a make-up examination for a missed examination, a student should have a documented or verifiable, and officially acceptable excuse. A student cannot get make-up examinations for two missed exams. The make-up examination for all exams will be after the final exam, and will include all topics.

**Lectures:**

|  |  |  |
| --- | --- | --- |
| **Section, Instructor** | **Lecture Time and Place** | **Instructor e-mail,**  **Office (Math building), office phone** |
| S1. Tolga Karayayla | Tue 8:40-10:30 (M13)  Thu 8:40-10:30 (M13) | [tkarayay@metu.edu.tr](mailto:tkarayay@metu.edu.tr)  222, (312) 210 5362 |
| S2. Özgür Kişisel | Tue 8:40-10:30 (M102)  Thu 8:40-10:30 (M102) | [akisisel@metu.edu.tr](mailto:akisisel@metu.edu.tr)  128, (312) 210 5388 |

**Office Hours:** To be announced.

**Important Dates:**

* **October 2:** Classes begin
* **October 9-13:** Add-drop period
* **October 29:** Republic Day (Sunday)
* **November 10:** Commemoration of Atatürk (Friday)
* **November 13-17:** Midterm 1 (date TBA)
* **December 4-10:** Course withdrawal applications
* **December 18-22:** Midterm 2 (date TBA)
* **January 1:** New Year’s Day (Monday)
* **January 5:** Classes end
* **January 8-20:** Final Exams
* **January 29:** Grades announced

**Course Schedule**

The table below is a rough guideline for the content of course lectures. Instructors may reorder their lectures as necessary/desired. Chapter and subsection titles are from the textbook, *Complex Variables and Applications*, Brown, J. W., Churchill, R. V., 8th ed. Each numbered box below indicates a 2 lecture-hour day, therefore there will be a total of 56 lecture-hours.

|  |  |  |
| --- | --- | --- |
| **Week 1:**  Oct.2-6 | 1 | **Chapter 1. Complex Numbers**  Sums and Products, Basic Algebraic Properties, Further Properties, Vectors and Moduli, Complex Conjugates |
| 2 | Exponential form, Products and Powers in Exponential Form, Arguments of Products and Quotients, Roots of Complex Numbers, Examples, Regions in the Complex Plane |
| **Week 2:**  Oct.9-13 | 3 | **Chapter 2. Analytic Functions**  Functions of a Complex Variable, Mappings, Mappings by the Exponential Function |
| 4 | Limits, Theorems on Limits, Limits Involving the Point at Infinity, Continuity |
| **Week 3:**  Oct.16-20 | 5 | Derivatives, Differentiation Formulas, Cauchy-Riemann Equations, Sufficient Conditions for Differentiability |
| 6 | Polar Coordinates, Analytic Functions, Examples, Harmonic Functions, Uniquely Determined Analytic Functions, Reflection Principle |
| **Week 4:**  Oct.23-27 | 7 | **Chapter 3. Elementary Functions**  The Exponential Function, The Logarithmic Function, Branches and Derivatives of Logarithms, Some Identities Involving Logarithms |
| 8 | Complex Exponents, Trigonometric Functions, Hyperbolic Functions, Inverse Trigonometric and Hyperbolic Functions |
| **Week 5:** Oct.30-Nov.3 | 9 | **Chapter 8. Mapping by Elementary Functions**  Linear Transformations, The Transformation w=1/z, Mappings by 1/z, Linear Fractional Transformations |
| 10 | An Implicit Form, Mappings of the Upper Half Plane, The Transformation w=sin z, Mappings by z2 and Branches of z1/2 |
| **Week 6:**  Nov.6-10 | 11 | Square Roots of Polynomials, Riemann Surfaces, Surfaces for Related Functions |
| 12 | **Chapter 4. Integrals**  Derivatives of Functions w(t), Definite Integrals of Functions w(t), Contours, Contour Integrals , Some Examples |
| **Week 7:**  Nov.13-17 | 13 | Examples with Branch Cuts, Upper Bounds for Moduli of Contour Integrals, Antiderivatives, Proof of the Theorem |
| 14 | Cauchy-Goursat Theorem, Proof of the Theorem, Simply Connected Domains, Multiply Connected Domains, Cauchy Integral Formula |
| **MIDTERM 1 (between Nov 13-17, date TBA)** | | |
| **Week 8:**  Nov.20-24 | 15 | An Extension of the Cauchy Integral Formula, Some Consequences of the Extension, Liouville’s Theorem and the Fundamental Theorem of Algebra, Maximum Modulus Principle |
| 16 | **Chapter 5. Series**  Convergence of Sequences, Convergence of Series, Taylor Series, Proof of Taylor’s Theorem |
| **Week 9:**  Nov.27-30 | 17 | Examples, Laurent Series, Proof of Laurent’s Theorem, Examples |
| 18 | Absolute and Uniform Convergence of Power Series, Continuity of Sums of Power Series, Integration and Differentiation of Power Series |
| **Week 10:**  Dec.4-8 | 19 | Uniqueness of Series Representations, Multiplication and Division of Power Series |
| 20 | **Chapter 6. Residues and Poles**  Isolated Singular Points, Residues, Cauchy’s Residue Theorem, Residue at Infinity |
| **Week 11:**  Dec.11-15 | 21 | The Three Types of Isolated Singular Points, Residues at Poles, Examples |
| 22 | Zeros of Analytic Functions, Zeroes and Poles, Behavior of Functions Near Isolated Singular Points |
| **Week 12:**  Dec.18-22 | 23 | **Chapter 7. Applications of Residues**  Evaluation of Improper Integrals, Example, Improper Integrals from Fourier Analysis |
| 24 | Jordan’s Lemma, Indented Paths, An Indentation Around a Branch Point, Integration Along a Branch Cut |
| **MIDTERM 2 (between Dec 18-22, date TBA)** | | |
| **Week 13:**  Dec.25-29 | 25 | Definite Integrals Involving Sines and Cosines, Argument Principle, Rouché’s Theorem |
| 26 | Inverse Laplace Transforms, Examples |
|  | **Holiday:** Jan 1st , Monday | |
| **Week 14:** Jan.1-5 | 27 | **Chapter 9. Conformal Mapping**  Preservation of Angles, Scale Factors, Local Inverses |
| 28 | Harmonic Conjugates, Transformations of Harmonic Functions, Transformations of Boundary Conditions |
| **FINAL EXAM (between Jan 8-20, date TBA)** | | |