Math 365 - Quiz 3

Name and Student ID:

Question: Find the largest $n \in \mathbb{Z}$ such that $288^n \mid 117 \cdot 118 \cdot 119 \cdots 289$.

$$288 = 2\cdot 144 = 2\cdot(12)^2 = 2\cdot 4^2\cdot 3^2 = 2^5\cdot 3^2$$

$$N = 117\cdot 118 \cdots 288\cdot 289 = \frac{289!}{116!}$$

Largest α such that $2^\alpha \mid 289!$

$$\alpha = \sum_{k=1}^{\infty}\left[\frac{289}{2^k}\right] = \left[\frac{289}{2}\right] + \left[\frac{289}{2^2}\right] + \cdots$$

$$= 144 + 72 + 36 + 18 + 9 + 4 + 2 + 1 = 286$$

Largest β such that $2^\beta \mid 116!$: $\beta = \sum_{k=1}^{\infty}\left[\frac{116}{2^k}\right] = 58 + 29 + 14 + 7 + 3 + 1 = 112$

Then $N = 2^{\alpha-\beta}\cdot M$ where $\gcd(2, M) = 1$

$$= 2^{286-112}\cdot M = 2^{174}\cdot M$$

Largest power of 3 dividing $289!$: $\gamma = \sum_{k=1}^{\infty}\left[\frac{289}{3^k}\right] = 96 + 32 + 10 + 3 + 1 = 142$

Largest power of 3 dividing $116!$: $\delta = \sum_{k=1}^{\infty}\left[\frac{116}{3^k}\right] = 38 + 12 + 4 + 1 = 55$

Then $N = 3^{\gamma-\delta}\cdot R$, where $\gcd(3, R) = 1$

$$= 3^{142-55}\cdot R = 3^{87}\cdot R$$

$N = 2^{174}\cdot 3^{87}\cdot S$ where $\gcd(2,S) = \gcd(3,S) = 1$

$$288^k = 2^{5k}\cdot 3^{2k} \mid N \iff 2^{5k}\cdot 3^{2k} \mid 2^{174}\cdot 3^{87}$$

$$\iff 5k \le 174 \wedge 2k \le 87$$

$$\iff k \le 34 \wedge k \le 43$$

$$\iff k \le 34$$

Maximum k such that $288^k \mid N = \frac{289!}{116!}$ is $\boxed{k = 34}$

Remark:

A Common Mistake In the Midterm 2 Answers:

Some of you solved a question in MT2 in the following way, which is WRONG!!!:

Question: Find largest k such that $9^k \mid \frac{26!}{12!}$ (for example)

You calculated:

$$\sum_{k=1}^{\infty}\left[\frac{26}{9^k}\right]=2 \qquad \sum_{k=1}^{\infty}\left[\frac{12}{9^k}\right]=1$$ and said Answer $=2-1=1$

WRONG!

(Look at $\sum_{k=1}^{\infty}\left[\frac{N}{p^k}\right]$ where p is a prime!!!!)

Or, some of you did:

$$\sum_{k=1}^{\infty}\left[\frac{26}{3^k}\right]=8+2=10, \quad \sum_{k=1}^{\infty}\left[\frac{12}{3^k}\right]=4+1=5$$

$\Rightarrow 3^{10} \mid 26!$

$9^5 \mid 26!$

$\Rightarrow 3^5 \mid 12!$

$9^2\cdot 3 \mid 12! \Rightarrow 9^2 \mid 12!$ (max power of 9 dividing 12!)

Then you wrote

$$\frac{9^5}{9^2}=9^3 \mid \frac{26!}{12!}$$ which is WRONG!!!

~~dividing the quo~~ Again, you must look at highest power of $\underline{3}$ (which is a prime) dividing the whole quotient $\frac{26!}{12!}$, then pass to power of $9=3^2$ dividing $\frac{26!}{12!}$ (~~Don't~~ look at highest power of 9 dividing 26! and 12! separately and take the difference!!!)

Correct Answer:

$$3^{10-5}=3^5 \mid \frac{26!}{12!}$$

so $9^k \mid \frac{26!}{12!} \Leftrightarrow 3^{2k} \mid \frac{26!}{12!}=3^5\cdot M$ where $\gcd(M,3)=1$

$\Leftrightarrow 2k\leq 5$

$\boxed{k\leq 2}$

max k is $\underline{2}$