M E T U
Department of Mathematics

	Elementary Number Theory I Midterm 2	

Code : *Math 365* Acad. Year : *2018-2019* Semester : *Fall* Instructor : *Tolga Karayayla* Date : *19.12.2018* Time : *17.40* Duration : *120 minutes*	Last Name : First Name : Student ID : Department : Signature : **7** Questions on **4** Pages SHOW DETAILED WORK!

1	2	3	4	5	6	7		

1. (10+10 pts.) a) Show that $\phi(3m) = 3\phi(n)$ if and only if $3|n$.

b) Show that $\sigma(n)$ is odd if and only if $n = k^2$ or $n = 2k^2$ for some integer k.

2. (5+10 pts.) Let ω be defined by $\omega(1) = 0$ and $\omega(n)$ is the number of distinct prime divisors of n for $n > 1$, $n \in \mathbb{Z}$.

a) Show that $f(n) = 2^{\omega(n)}$ is a multiplicative function from $\mathbb{Z}^+$ to $\mathbb{Z}$.

b) Show that $\tau(n^2) = \sum_{d|n} 2^{\omega(d)}$ for all $n \in \mathbb{Z}^+$.

3. (10 pts.) Find a formula for $\sum_{d|n} \frac{(\mu(d))^2}{\phi(d)}$ in terms of the prime factorization of n.

4. (10+10 pts.) a) Find the largest $k \in \mathbb{Z}$ such that $9^k | \frac{300!}{(100!)^3}$.

b) Find the largest $k \in \mathbb{Z}$ such that $175^k | 365 \cdot 364 \cdot 363 \cdots 102 \cdot 101$.

5. (10 pts.) If $n > 1$ is a composite integer, show that $\phi(n) \leq n - \sqrt{n}$. (Hint: Consider the smallest prime divisor p of n.)

6. (10 pts.) Find $0 \leq x \leq 359$ such that $7^{9700} \equiv x \pmod{360}$.

7. (3×5 pts.) State whether the following statements are True or False. Give brief explanation for each case.

a) Let $n > 1$. If $a^n \equiv a \pmod{n}$ for all $a \in \mathbb{Z}$, then n is a prime.

b) Let $n > 1$. If $(n-1)! \equiv -1 \pmod{n}$, then n is a prime.

c) If $\sum_{d|n} f(d) = \sum_{d|n} g(d)$ for all $n \in \mathbb{Z}^+$, then $f(n) = g(n)$ for all $n \in \mathbb{Z}^+$.