

M E T U

Department of Mathematics

					Discrete Mathematics									
					Midterm II									
Code : <i>Math 112</i>					Last Name :									
Acad. Year : <i>2017-2018</i>					Name :									
Semester : <i>Spring</i>					Department :									
Instructor : <i>Finashin, Emelyanov</i>					Signature :									
Date : <i>03.05.2018</i>					5 Questions on 4 Pages SHOW YOUR WORK!									
Time : <i>17.40</i>														
Duration : <i>110 minutes</i>														
1	2	3	4	5										

1.(8+12 pts.) Suppose that a_n is a sequence which satisfies a linear homogeneous recurrence relation with constant coefficients. Assume that the characteristic equation of this recurrence relation is

$$(r - 2)^2(r - 3) = 0.$$

a) Find the recurrence relation of a_n .

b) Find an explicit formula for a_n if $a_0 = 7$, $a_1 = 25$, $a_2 = 77$.

2. (15+5 pts.) Let a_n be the number of strings (words, not necessarily meaningful) of length n consisting of the letters A, B, C, D, E that do *not* contain two consecutive A's. (Note that these strings may not contain some of the 5 letters.)

a) Find a recurrence relation that a_n satisfies.

b) Compute a_1 , a_2 , a_3 , a_4 and a_5 .

3. (10+10 pts.) a) A monkey types up a random word of length 3 on a keyboard with 26 letters. Find the probability that the word contains an odd number of vowels (Vowels are A,E,I,O and U).

3.b) Another monkey picks randomly 3 letters from a box containing the 26 letters, without replacement (i.e. once a letter is picked, it is not returned into the box). Find the probability that these letters contain an even number (including 0) of vowels. (e.g. $\{A, B, E\}$ is such a selection).

4. (10+10 pts.) Suppose that a box contains 3 white and 1 black balls, and a second box contains 2 white and 3 black balls. Suppose also that a person chooses at random one of the boxes, then chooses one of the balls in it.

a) Find the probability that the chosen ball is black.

b) Suppose that the chosen ball turned out to be black. Find the probability that this black ball has been selected from the first box.

5. (10+10 pts.) A die is biased such that its outcomes have the following probabilities:

$$P(1) = P(2) = P(4) = P(5) = P(6), \text{ and } P(3) = \frac{1}{4}.$$

a) Suppose that this biased die is rolled six times consecutively. What is the probability that in exactly four of these six rolls an even number comes up?

b) Suppose that together with this biased die a second die, which is fair, is also rolled. If these two dice are rolled once, what is the probability that one of the two outcomes is 3 and the other is 6?