

# M E T U

## Department of Mathematics

Elementary Number Theory I						
Midterm 1						
Code : <i>Math 365</i>			Last Name :			
Acad. Year : <i>2018-2019</i>			First Name :		Student ID :	
Semester : <i>Fall</i>			Department :			
Instructor : <i>Tolga Karayayla</i>			Signature :			
Date : <i>21.11.2018</i>			<b>7 Questions on 4 Pages</b> <b>SHOW DETAILED WORK!</b>			
Time : <i>17.40</i>						
Duration : <i>120 minutes</i>						
1	2	3	4	5	6	7

1. (10+5 pts.) a) Apply Euclidean Algorithm to calculate  $d = \gcd(9269, 2249)$  and find  $x, y \in \mathbb{Z}$  such that  $d = 9269x + 2249y$ .

b) Find all solutions  $(x, y) \in \mathbb{Z}^2$  of the Diophantine equation  $9269x + 2249y = 314$ .

2. (7 pts.) Suppose that  $n > 1$  is an integer and  $n$  is not divisible by any prime  $p \leq \sqrt[3]{n}$ . Show that either  $n$  is a prime or  $n$  is a product of (not necessarily distinct) two primes.

3. (20 pts.) Solve the system of congruences

$$x \equiv 8 \pmod{14}$$

$$x \equiv 4 \pmod{39}$$

$$x \equiv 5 \pmod{55}$$

4. (15 pts.) Find all solutions modulo 642 of the congruence  $198x \equiv 156 \pmod{642}$

5. (15 pts.) Solve the system of congruences

$$x \equiv 75 \pmod{216}$$

$$x \equiv 139 \pmod{400}$$

$$x \equiv 164 \pmod{405}$$

6. (7+7 pts.) a) Find  $0 \leq x \leq 18$  such that  $6^{229} \equiv x \pmod{19}$ .

b) Let  $n = 2p$  where  $p \geq 3$  is a prime. Show that  $a^{n-1} \equiv a \pmod{n}$  for all  $a \in \mathbb{Z}$ .

7. (7+7 pts.) a) Show that if  $\gcd(a, b) = 1$ ,  $c|a$  and  $d|b$ , then  $\gcd(c, d) = 1$ .

b) If  $\gcd(a, b) = p$  where  $p$  is a prime, then show that  $\gcd(a^3, b^4)$  is either  $p^3$  or  $p^4$ .