

M E T U

Department of Mathematics

Discrete Mathematics		
Midterm I		
Code : <i>Math 112</i>	Last Name :	
Acad. Year : <i>2017-2018</i>	Name :	
Semester : <i>Spring</i>	Department :	Student No :
Instructor : <i>Emelyanov, Finashin, Karayayla, Önal, Seven.</i>	Signature :	
Date : <i>29.03.2018</i>	5 Questions on 4 Pages	
Time : <i>17.40</i>	SHOW DETAILED WORK!	
Duration : <i>120 minutes</i>		
1	2	3
4	5	

1. (8+12 pts) Suppose that a mathematics department offers 8 courses each semester (the same courses are offered each time), and these courses are distributed between 4 instructors so that each one teaches two courses every semester.

a) In how many ways can the courses be assigned to the instructors in one semester?

b) In how many ways can the courses be assigned to the instructors in two semesters so that for each instructor the pair of courses he teaches in one semester is not the same as in the other semester (at least one course must be different)?

2. (2×10 pts) In how many ways can the 10 numerals $0, 1, \dots, 9$ and the 26 letters A, \dots, Z be permuted (arranged in a row) in each of the following cases?

a) The numerals appear in the increasing order from left to right (there may be letters between numerals).

b) There is at least one letter between any pair of numerals.

3. (2×10 pts) **a)** Find the coefficient of x^5 in the expansion of $(x^2 + x^{-1} - 2)^{10}$.

3.b) Prove that for any integer $0 \leq k \leq 20$ the following identity holds:

$$\binom{k}{k} + \binom{k+1}{k} + \binom{k+2}{k} + \cdots + \binom{20}{k} = \binom{21}{k+1}.$$

4. (2×10 pts) A collection of letters consists of A,A,B,B,C,C,D,E,F,G,H,I,J,K,L,M,N,O (totally 18 letters, containing 3 pairs of identical letters).

a) In how many ways can these letters be arranged around a circle? (Arrangements which differ by a circular rotation in the plane are considered to be the same.)

b) In how many ways can these 18 letters be split into 3 groups of equal size so that for each of the 3 pairs of identical letters, both of the identical letters appear in the same group (but for different pairs, the groups containing them may be different).

5. (6+7+7 pts) 20 identical diamonds are distributed to 8 people. In how many ways can this be done in each of the following cases?

a) Each person gets at least two diamonds.

b) 6 people get an odd number of diamonds and 2 people get an even number of diamonds.

c) Nobody gets exactly 6 diamonds.