M E T U
Department of Mathematics

	Complex Calculus Midterm 2	

Code : *Math 353* Acad. Year : *2017-2018* Semester : *Fall* Date : *December.18.2018* Time : *17:40* Duration : *120 minutes*	Last Name : Name : Student No. : Department : Section : Signature : 6 QUESTIONS ON 4 PAGES TOTAL 100 POINTS

1	2	3	4	5	6	**SHOW YOUR WORK**	

Question 1 (12 pts) Show that if $f(z)$ is entire and $|f(z)| \leq |e^z|$ for all $z \in \mathbb{C}$, then there exists a constant $c \in \mathbb{C}$ such that $f(z) = ce^z$.

Question 2 (13 pts) Let $f(z)$ be a function which is continuous on a closed and bounded region $R \subset \mathbb{C}$, and suppose that f is analytic and non-constant throughout the interior of R. Assuming that $f(z) \neq 0$ anywhere in R, prove that $|f(z)|$ has an absolute minimum value m on R which occurs on the boundary of R and never in the interior of R.

Question 3 (6+6+6+7=25 pts) Evaluate the following contour integrals where $\mathcal{C}$ is the positively oriented boundary of the given region R

a) $\displaystyle\int_{\mathcal{C}} \frac{\cos z}{z(z^2+8)}dz,\ R = \{x+iy|\,|x| \leq 2,\ |y| \leq 2\}.$

b) $\displaystyle\int_{\mathcal{C}} \frac{\cosh z}{z^4}dz$, the same R as in part (a).

c) $\displaystyle\int_{\mathcal{C}} \frac{z}{(z-1+i)^2}dz,\ R = \{z \in \mathbb{C}|\,|z-2-2i| \leq \tfrac{3}{2}\}.$

d) $\displaystyle\int_{\mathcal{C}} \frac{1}{(z^2+1)(z^2-2z-3)}dz,\ R = \{z \in \mathbb{C}|\,|z| \leq 2,\ 0 \leq arg(z) \leq 5\pi/4\}.$

Question 4(7+8=15 pts) Let $f(z) = \log z = \ln r + i\theta$, where $-\pi/4 < \theta < 7\pi/4$, and let $\mathcal{C}$ be the part of the graph of the polar equation $r = 1+\sin(2\theta/3)$ where $0 \leq \theta \leq 3\pi/2$. $\mathcal{C}$ is given a direction such that its initial point is 1.

a) Express $\int_{\mathcal{C}} f(z)dz$ as a definite integral using a parametrization of $\mathcal{C}$.

b) Evaluate $\int_{\mathcal{C}} f(z)dz$ without using a parametrization of the given contour $\mathcal{C}$.

Question 5 (15 pts) Find the Taylor series expansion of $f(z) = \dfrac{\cos(z)}{z}$ around the point $z_0 = \pi$ up to and including the $(z-\pi)^3$ term. What is the radius of convergence of this series? (i.e. find the largest $R > 0$ such that $f(z)$ is equal to this Taylor series on the open disk $|z - \pi| < R$.)

Question 6 (6+7+7=20 pts) Find the Laurent series expansions of the function $f(z) = \dfrac{z^2}{(z-2)(z+3)}$ centered at $z_0 = 0$, where the representation is valid;

(a) in the region $0 \leq |z| < 2$,

(b) in the region $2 < |z| < 3$,

(c) in the region $3 < |z|$.