M E T U
Department of Mathematics

	Complex Calculus MidTerm 1	

Code : *Math 353* Acad. Year : *2017-2018* Semester : *Fall* Date : *November.13.2017* Time : *17:40* Duration : *120 minutes*	Last Name : Name : Student No. : Department : Section : Signature : 6 QUESTIONS ON 4 PAGES TOTAL 100 POINTS

1	2	3	4	5	6	**SHOW YOUR WORK**	

Question 1 (10+10=20 pts) a) Show that for any complex numbers z and w, $Re(z) > 0$ and $Re(w) > 0$ implies that $Arg(zw) = Arg(z) + Arg(w)$. ($Re(z)$: real part of $z \in \mathbb{C}$. $Arg(z)$: principal argument of $z \in \mathbb{C}$.)

b) Assume that $\lim_{z \to z_0} f(z) = 0$ and there exists $M > 0$ ($M \in \mathbb{R}$) such that $|g(z)| < M$ for all z in some neighborhood of z_0. Show that $\lim_{z \to z_0} f(z)g(z) = 0$.

Question 2 (6+6+6=18 pts) Let $f(z) = 3x^3 - 2y^3 + x^2 + y^2 + i(3x^2y + 2xy^2)$ where $z = x + iy$ ($x,\ y \in \mathbb{R}$).

a) Find all points $z \in \mathbb{C}$ such that $f'(z)$ exists.

b) Find all points $z \in \mathbb{C}$ such that $f'(z) = 0$.

c) If it exists, determine the largest set $D \subset \mathbb{C}$ such that $f(z)$ is analytic on D.

Question 3 (14 pts) Let $V(x, y) = 5x^4y - 10x^2y^3 + y^5 + 2xy$. Find a function $U(x, y)$ (if it exists) such that $F(z) = U(x, y) + iV(x, y)$ is an entire function, where $z = x + iy$ such that $x, y \in \mathbb{R}$.

Question 4 (8+10=18 pts) (a) Prove the identities $\cosh^2 x - \sinh^2 x = 1$ and $\sinh x + \cosh x = e^x$ for $x \in \mathbb{R}$ by using the definitions of the functions directly.

(b) Deduce that $\cosh^2 z - \sinh^2 z = 1$ and $\sinh z + \cosh z = e^z$ for all $z \in \mathbb{C}$ from part (a) by using the theorem on uniqueness of analytic extensions (analytic continuation).

Question 5 (13 pts) Suppose that $f(z)$ is analytic on a domain (an open and connected set) $D \subseteq \mathbb{C}$ and for all $z \in D$, we have

$$Re(f(z)) = 2Im(f(z)).$$

Prove that $f(z)$ must be constant on D.

Question 6 (17 pts) Let $f(z)$ be the linear fractional transformation such that $f(0) = 1$, $f(1) = i$ and $f(i) = 0$. What is the image of the unit circle under f?