M E T U
Department of Mathematics

<table>
<tr><td></td><td colspan="2">Discrete Mathematics
FINAL EXAM</td><td></td></tr>
<tr><td colspan="2">Code : Math 112
Acad. Year : 2017-2018
Semester : Spring
Instructor : Emelyanov, Finashin, Karayayla, Önal, Seven.
Date : 24.05.2018
Time : 09.30
Duration : 120 minutes</td><td colspan="2">Last Name :
Name : Student No:
Department :
Signature :

5 Questions on 4 Pages
Total 100 Points</td></tr>
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1	2	3	4	5		

Q1. (10 + 10 **pts.**) **a)** A $2 \times n$ rectangle is covered by smaller rectangles of dimensions 2×1 and 2×2. If the small pieces of rectangles can be red or yellow in color, find a recursion relation for a_n where a_n is the number of ways of covering the large rectangle with these small rectangles. (Give enough number of initial values. Do not solve the recurrence relation.)

b) Letters of the word **CLASSROOM** are shuffled randomly. Find the probability that two identical letters appear consecutively.

Q2.(10 + 10 **pts.**) **a)** Let $A \subseteq X = \{1, 2, ..., 20\}$ where $|A| \geq 12$. Show that there are two elements $a, b \in A$ such that $a + b = 19$.

b) Let $B \subseteq Y = \{1, 2, ..., 112\}$ where $|B| = 85$. Show that there are at least 4 consecutive numbers in B.

Q3.(10 + 10 **pts.**) **a)** A student reads a book of 100 pages in 9 days by reading at least one page everyday. Assuming that a whole number (integer) of pages are read each day, show that there are two consecutive days in which the student reads at least 21 pages in total.

Q3.b) Prove that any connected graph (with at least two vertices) which has no loops and no multiple edges has a pair of vertices u and v such that $deg(u) = deg(v)$ (degrees of the two vertices are equal).

Q4.($10+10$ **pts.**) **a)** For each pair of the 3 graphs below, show whether they are isomorphic or not.

b) Sketch at least 5 non-isomorphic trees with 6 vertices. (No explanation is asked, only sketch the trees.)

Q5.$(5 \times 4$ **pts.)** **a)** If it exists, sketch a graph G_1 which has 7 vertices $\{v_1, v_2, ...v_7\}$ and 20 edges, and the degrees of the vertices are given by $deg(v_k) = k$ for $k = 1, 2, ..., 7$.

b) Sketch a simple graph (a graph with no loops and no multiple edges) G_2 which has 5 vertices with degrees $deg(v_1) = 4$, $deg(v_2) = 3$, $deg(v_3) = deg(v_4) = 2$, and $deg(v_5) = 1$.

c) Does the graph G_2 in part **(b)** have an Euler circuit? If so, show the circuit by numbering its edges as $e_1, e_2, ...$ etc. (e_1 is the first edge of the circuit, e_k is the kth edge.)

d) Does the graph G_2 in part **(b)** have an Euler path? If so, show the path by numbering its edges as $e_1, e_2, ...$ etc. (e_k is the kth edge of the path.)

e) Is the graph G_2 in part **(b)** a bipartite graph?