M E T U
Department of Mathematics

	Introduction to Basic Algebra Structures FINAL	
Code : *Math 116* Acad. Year : *2013-2014* Semester : *Spring* Instructor : *G.E., T.K., M.K., A.S.* Date : *06.06.2014* Time : *09:30* Duration : *120 minutes*	Last Name : Name : Student No : Department : Signature : **5** Questions on **4** Pages Total **80** Points	

1	2	3	4	5	6		

1.(15 pts.) It is given that the set $R = \{ \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \mid a, b, c \in \mathbb{Z} \}$ is a ring with respect to matrix addition and matrix multiplication. Show that $I = \{ \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix} \mid a, b \in \mathbb{Z} \}$ is an ideal of R.

2.(15 pts.) Let $R = \{ \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \mid a, b, c \in \mathbb{Q} \}$ and $S = \mathbb{Q}$ (the set of rational numbers). Define the map $\alpha : R \longrightarrow S$
by setting $\alpha(\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}) = a - c$. Is α a ring homomorphism? Explain your reasoning.

3. (15 pts.) Let $R = \{[0], [2], [4], [6], [8]\} \subset \mathbb{Z}_{10}$. It is given that R is a ring under addition and multiplication modulo 10.

(i) Find the unity (multiplicative identity) of R, if any.

(ii) Is R an integral domain? Explain why.

(iii) Is R a field? Explain why.

4. (15 pts.) In $\mathbb{Z}_5[x]$, let $f(x) = 3x^5 - 2x^4 - x^3 - x + 1$ and $g(x) = 2x^2 + 3x + 1$.

(i) Find polynomials $q(x)$ and $r(x)$ in $\mathbb{Z}_5[x]$ such that $f(x) = q(x)g(x) + r(x)$ where the degree of $r(x)$ is at most 1.

(ii) Find $gcd(f(x), g(x))$.

5. (20 pts.) Let $G = \mathbb{Z}_{20}$ and let $H = \langle [4] \rangle$.

(i) Find the distinct left cosets of the subgroup H in the group G.

(ii) Find the order of the element $[6] + H$ in the quotient group G/H.

(iii) Is G/H cyclic? Explain why.