Math 120 - 20152 - Recitation 5

1) Write down the equation of the plane which contains the line of intersection of the two planes $x + y + 2z = 4$ and $3x - 2y + z = 6$, and
a) which also passes through $(0, 1, 0)$.
b) which is parallel to the line $L : x - 1 = \frac{y-2}{3} = \frac{3-z}{2}$.

2) Write down the equation of the line L_1 which passes through $(1, 3, 5)$ and which intersects both of the lines
$L_2 : \vec{r_2}(t) = (6 + 3t, 2 + t, 11 + 2t)$, $t \in \mathbb{R}$ and
$L_3 : \vec{r_3}(t) = (5 - 7t, 4 + 2t, -t)$, $t \in \mathbb{R}$.

3) Find the intersection point of the following two lines if they intersect.

$$L_1 : \begin{cases} x = 1 + 2t \\ y = 7 - t \\ z = 6 - 2t \end{cases} \quad t \in \mathbb{R} \qquad L_2 : \begin{cases} x = 12 - 3t \\ y = -7 + 4t \\ z = 5 - t \end{cases} \quad t \in \mathbb{R}.$$

And if they intersect, write down the equation of the plane which contains both of the lines.

4) a) Show that the following two lines are skew.
$L_1 : \vec{r_1}(t) = (1 + 2t, 2 - t, 4 + 3t), t \in \mathbb{R}$
$L_2 : \vec{r_2}(t) = (3 + 3t, 6 - 4t, 1 + 5t), t \in \mathbb{R}$.
b) Find the distance from L_1 to L_2 (use the formula).
c) Find the two points $P \in L_1$ and $Q \in L_2$ such that the distance $|PQ|$ is minimum.
d) Write down the equation of the line L which intersects both of the lines L_1 and L_2 perpendicularly.

5) Let $A = (1, 0, 2)$ and $B = (3, 6, 8)$ be two points. Write down the equation of the line L_1 which intersects the line segment $[AB]$ perpendicularly at the midpoint and which also intersects the line $L_2 : 3 - x = \frac{y}{4}, z = 4$.

6) Calculate the distance from the point $(1, 2, 3)$ to the plane $x + 3y - 4z = 2$ and find the point P on the plane which is closest to $(1, 2, 3)$.

7) Calculate the distance from the point $P(2, 3, 4)$ to the line $L : \vec{r}(t) = (1 + t, 2 - t, 3 + 2t), t \in \mathbb{R}$ and find the point $Q \in L$ which is closest to P.

8) Write down the equations of all lines (if any exists) which intersect the line $L : \vec{r}(t) = (1 + t, 2 + t, 3 + t), t \in \mathbb{R}$ with an angle of $\pi/6$ radians at the point $(2, 3, 4)$ and which are parallel to the plane $2x - 3y + 3z = 120$. How many such lines are there? (Hint: focus on a direction vector $\vec{v} = (a, b, c)$ of unit length for the line. 3 equations in 3 variables a, b and c. Answer: 2 lines. Note: solution of the equations looks nasty, don't write the final form of the line equation.)