Math 120-Recitation Week 6

1. Show that the lines $L_1 : x = t,\ y = t,\ z = t$ and $L_2 : x = -1 + s,\ y = 2s,\ z = 3s$ are skew, and find the distance between these lines.

2. Find an equation of the plane that contains the point $(-2, 1, 4)$ and $(0, 3, 1)$ and contains a line parallel to the vector $2\vec{\mathbf{i}} - 4\vec{\mathbf{j}} + 6\vec{\mathbf{k}}$.

3. Let L be the line given by the parametric equations $x = 1+2t,\ y = -1+3t,\ z = -5 + 7t, \quad t \in \mathbb{R}$, and let M be the plane $2(x-1) + 2(y+3) - z = 0$. Find two points on the line L at a distance 3 from the plane M.

4. Identify the surfaces represented by the equations below and sketch their graphs.

a) $16x^2 = y^2 + 4z^2$

b) $x^2 + y^2 + z^2 - 6y + 2z + 7 = 0$

5. Consider the conical helix given by $\vec{\mathbf{r}}(t) = t\cos t\vec{\mathbf{i}} + t\sin t\vec{\mathbf{j}} + t\vec{\mathbf{k}}, \quad 0 \le t \le 2\pi$. (Why is this called conical helix?)

a) Find the equation of the tangent line to the helix at the point $(-\pi, 0, \pi)$.

b) Find the length of the helix.

6. At time $t = 0$, a particle is located at the point $(1, 2, 3)$. It travels in a straight line to the point $(4, 1, 4)$, has speed 2 at $(1, 2, 3)$ and constant acceleration $3\vec{\mathbf{i}} - \vec{\mathbf{j}} + \vec{\mathbf{k}}$. Find an equation for the position vector $\vec{\mathbf{r}}(t)$ of the particle at time t.