M E T U
Department of Mathematics

1.(20 pts.) Consider the points $A(3, 1)$ and $B(7, -5)$ in the Cartesian plane.

a) Find the equation of the line which passes through A and B.

b) Find the midpoint of the line segment $\overline{AB}$.

c) Find the equation of the perpendicular bisector of $\overline{AB}$, i.e. the line that intersects the line segment $\overline{AB}$ at its midpoint with a right angle.

2.(20 pts.) Consider the polar equation $r = 2sin(2\theta)$ for $0 \leq \theta \leq \pi$.

a) Find the values of r for the following values of $\theta = 0, \frac{\pi}{12}, \frac{\pi}{8}, \frac{\pi}{6}, \frac{\pi}{4}, \frac{2\pi}{6}, \frac{3\pi}{8}, \frac{5\pi}{12}$.

b) Sketch the graph of the equation $r = 2sin(2\theta)$ for $0 \leq \theta \leq \pi$.

3.   (20 pts.)   Consider the points $A(0,4)$, $B(6,0)$ and the line L with equation $y = 2x + 1$ in the Cartesian plane. Find all points P on L such that the triangle APB is a right triangle (with right angle being at vertex P).

3. (20 pts.)

a) Show that the points $A(-1, 3)$, $B(3, 11)$ and $C(5, 15)$ are collinear (i.e. they lie on a line in the Cartesian plane).

b) Find a unit vector $\vec{u}$ that has the same direction as $8\vec{i} - \vec{j} + 4\vec{k}$.

c) Find the angle between the vectors $\vec{i} + 2\vec{j} - 2\vec{k}$ and $\vec{i} - \vec{k}$.

4. **(20 pts.)** Assume that $\bar{x}\bar{y}$ coordinate system is obtained from the xy-coordinate system by a rotation through angle $\alpha = \tan^{-1}(3/4)$.

a) Find $\cos(\alpha)$ and $\sin(\alpha)$.

b) Write x and y in terms of $\bar{x}$ and $\bar{y}$.

c) Let L be the line with xy-equation $3x - 4y + 50 = 0$. Find the equation of L in the $\bar{x}\bar{y}$ coordinate system.

d) Let P be the point whose xy-coordinates are $(4, 3)$. Find the $\bar{x}\bar{y}$ coordinates of P.

e) Find the distance from the point P (given in part d) to the $\bar{y}$-axis.