M E T U
Department of Mathematics

	Analytic Geometry **Final Exam**	

Code : *Math 115* Acad. Year : *2017-2018* Semester : *Fall* Coordinator : *E. Coskun* Date : *13.1.2018* Time : *13:30* Duration : *120 minutes*	Last Name : Name : Student No : Department : Signature : **5** Questions on **4** Pages Total **100** Points

1	2	3	4	5		

1. (**5+5+5+5 pts.**) Consider the hyperbola whose foci are $F(4,4)$, $F'(-4,-4)$ and eccentricity is $e=2$.

(a) Find the center C.

(b) Write an equation for the axis ℓ of the hyperbola.

(c) Find the intersections G and G' of ℓ with the directrices d and d', respectively.

(d) Find the xy-equation of the hyperbola.

2. (**3+2+3+6+6 pts.**) Let S be a surface with the equation $4(x-1)^2+9(z-3)^2=(y-4)^2$.
(a) Identify the surface S. What type of a quadratic surface is S? (Explain).

(b) Describe the plane section of S by the plane $\mathcal{P}_1 : y = 4$, i.e. find $S \cap \mathcal{P}_1$.

(c) Describe the plane section of S by the plane $\mathcal{P}_2 : x = 1$.

Write the type and find the eccentricity e of the conic $\mathcal{C}$ which is the intersection of the given surface with each of the following planes (In each case write the equation of the conic in its simplest form):
(d) $y = 6$

(e) $z = 4$

3. (**14+6 pts.**) Let S be a conic with the equation $2x^2+2\sqrt{2}xy+3y^2=1$.
(a) Find $\cos\alpha$ and $\sin\alpha$ such that $0<\alpha<\pi/2$ and when xy-coordinate system is rotated by α radians to obtain $\bar{x}\bar{y}$-coordinate system, the equation of S has no $\bar{x}\bar{y}$-term. Moreover write down x and y in terms of $\bar{x}$ and $\bar{y}$.

(b) Write down the equation of S in terms of the $(\bar{x}, \bar{y})$ coordinates.

4. (**6+7+7 pts.**) Let ℓ be a line and $\mathcal{P}$ be a plane in 3-space. P and Q are said to be symmetric (partners of each other) about the line ℓ if ℓ is a perpendicular bisector of the segment $[PQ]$. P and Q are said to be symmetric (partners of each other) about the plane $\mathcal{P}$ if $\mathcal{P}$ is perpendicular to the segment $[PQ]$ and bisects it. Let $P(4, 2, 8)$ be a point. Find the symmetric partner Q of P with respect to
(a) the point $M(2, 4, 6)$

(b) the plane $\mathcal{P} : x = 10$

(c) the line $\ell : (x, y, z) = (4, 2, 4) + t(1, 1, 0)$ for $t \in \mathbb{R}$.

5. (**10+10 pts.**)

(a) Write the equation of the ellipse with foci $F_1(3,0),\ F_2(-3,0)$ and minor axis (diameter) 8 units long.

(b) Find the value(s) of m so that the plane $\mathcal{P} : x + z = m$ touches (i.e. intersects at one point) the sphere $x^2 + y^2 + z^2 = 4$.