

Quiz 5

Evaluate $\lim_{x \rightarrow 0^+} \frac{1}{x^{3/2}} \int_0^{\sqrt{x}} \tan(t^2) dt$

Solution

$$\lim_{x \rightarrow 0^+} \frac{1}{x^{3/2}} \int_0^{\sqrt{x}} \tan(t^2) dt = \lim_{x \rightarrow 0^+} \frac{\int_0^{\sqrt{x}} \tan(t^2) dt}{x^{3/2}} \left[\frac{0}{0} \right]$$

L'Hopital Rule

$$= \lim_{x \rightarrow 0^+} \frac{\frac{d}{dx} \left(\int_0^{\sqrt{x}} \tan(t^2) dt \right)}{\frac{d}{dx} (x^{3/2})}$$

FTC

$$\downarrow$$
$$= \lim_{x \rightarrow 0^+} \frac{\tan(\sqrt{x})^2 \cdot \frac{d}{dx} \sqrt{x}}{\frac{3}{2} \sqrt{x}} = \lim_{x \rightarrow 0^+} \frac{\tan x \cdot \frac{1}{2\sqrt{x}}}{\frac{3}{2} \sqrt{x}}$$

$$= \lim_{x \rightarrow 0^+} \frac{\tan x}{3x} = \frac{1}{3} \lim_{x \rightarrow 0^+} \frac{\sin x}{x} \cdot \frac{1}{\cos x} = \frac{1}{3}$$