Math 119 T2 Quiz 3 (03.12.2020)

Let g be a function satisfying $g(1) = 1$ and $g'(1) = 4$.

If $f(x) = (g(x))^{\arctan x}$, find the slope of tangent line to $f(x)$ at $x = 1$.

$g'(1) = 4$, since g is differentiable at $x=1$
$g(x)$ is continuous at $x=1$, $g(1)=1$ so
$\lim_{x \to 1} g(x) = 1 \to g(x)$ takes positive values (near 1 since limit is 1)
around 1.

$$f(x) = e^{\ln(g(x))^{\arctan x}} = e^{\arctan x \cdot \ln(g(x))}$$

$$f'(x) = \underbrace{e^{\arctan x \cdot \ln(g(x))}}_{= f(x)} \cdot \left(\frac{1}{1+x^2} \cdot (\ln(g(x))) + \arctan x \cdot \frac{g'(x)}{g(x)} \right)$$

$$f'(1) = \underbrace{e^{\arctan 1 \cdot \ln(\underbrace{g(1)}_{1})}}_{1} \cdot \left(\frac{1}{2} \overbrace{\ln(g(1))}^{0} + \underbrace{\arctan 1}_{\frac{\pi}{\cancel{4}}} \cdot \frac{\cancel{g'(1)}^{4}}{\underbrace{g(1)}_{1}} \right)$$

$= \pi \to$ slope of the tangent line of $f(x)$
at $x = 1$ $(= f'(1))$