

MATH 119 T-1 QUIZ 3 (10.12.2020)

Question: Evaluate $\lim_{x \rightarrow 0} \left(\frac{1}{\arcsin x} - \frac{1}{x} \right)$

Solution:

$\lim_{x \rightarrow 0} \left(\frac{1}{\arcsin x} - \frac{1}{x} \right)$ [$\infty - \infty$] form
 (equate denominators)

$= \lim_{x \rightarrow 0} \frac{x - \arcsin x}{x \cdot \arcsin x}$ [$\frac{0}{0}$ form]

$\stackrel{LH}{=} \lim_{x \rightarrow 0} \frac{1 - \frac{1}{\sqrt{1-x^2}}}{1 \cdot \arcsin x + x \cdot \frac{1}{\sqrt{1-x^2}}}$

equate denominators
 on the upper & lower side

$= \lim_{x \rightarrow 0} \frac{\frac{\sqrt{1-x^2} - 1}{\sqrt{1-x^2}}}{\frac{\sqrt{1-x^2} \arcsin x + x}{\sqrt{1-x^2}}} = \lim_{x \rightarrow 0} \frac{\sqrt{1-x^2} - 1}{\sqrt{1-x^2} \arcsin x + x}$ [$\frac{0}{0}$] form

$\stackrel{LH}{=} \lim_{x \rightarrow 0} \frac{\frac{-2x}{2\sqrt{1-x^2}}}{\frac{-2x}{2\sqrt{1-x^2}} \arcsin x + \sqrt{1-x^2} \cdot \frac{1}{\sqrt{1-x^2}} + 1}$

$= \lim_{x \rightarrow 0} \frac{\frac{-x}{\sqrt{1-x^2}}}{\frac{x}{\sqrt{1-x^2}} \arcsin x + 2} = 0$

Question: Evaluate $\lim_{x \rightarrow 0} \left(\frac{1}{\arcsin x} - \frac{1}{x} \right)$ MATH 119T-1 QUIZ 3 (10.12.2020)

Solution 2 $\lim_{x \rightarrow 0} \left(\frac{1}{\arcsin x} - \frac{1}{x} \right) \rightarrow \infty - \infty$ form
(equating denominators)

$$= \lim_{x \rightarrow 0} \frac{x - \arcsin x}{x \cdot \arcsin x} \stackrel{[0/0] \text{ form}}{=} \lim_{x \rightarrow 0} \frac{1 - \frac{1}{\sqrt{1-x^2}}}{1 \cdot \arcsin x + \frac{x}{\sqrt{1-x^2}}}$$

$$\stackrel{[0/0] \text{ form}}{=} \lim_{x \rightarrow 0} \frac{-2x}{2\sqrt{(1-x^2)^3}} + \frac{1}{\sqrt{1-x^2}} + \frac{1 \cdot \sqrt{1-x^2} + x \cdot \frac{+2x}{2\sqrt{1-x^2}}}{(\sqrt{1-x^2})^2}$$

quotient rule
be careful.

$$= \lim_{x \rightarrow 0} \frac{\frac{-x}{\sqrt{(1-x^2)^3}}}{\frac{1}{\sqrt{1-x^2}} + \frac{1-x^2+x^2}{\sqrt{(1-x^2)^3}}}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{-x}{\sqrt{(1-x^2)^3}}}{\frac{1-x^2+1}{\sqrt{(1-x^2)^3}}} = \lim_{x \rightarrow 0} \frac{-x}{2-x^2} = 0$$

$$\left(\left(\frac{1}{\sqrt{1-x^2}} \right)' = \left((1-x^2)^{-1/2} \right)' = \frac{1}{2} (-2x) (1-x^2)^{-3/2} = \frac{-x}{\sqrt{(1-x^2)^3}} \right)$$

Question: Evaluate $\lim_{x \rightarrow 0} \left(\frac{1}{\arcsin x} - \frac{1}{x} \right)$ MATH 119T-1 QUIZ 3 (10.12.2020)

Solution 3 $\lim_{x \rightarrow 0} \left(\frac{1}{\arcsin x} - \frac{1}{x} \right) \rightarrow \infty - \infty$ form
 equate the denominators

$$= \lim_{x \rightarrow 0} \frac{x - \arcsin x}{x \cdot \arcsin x} = \lim_{x \rightarrow 0} \frac{x \left(1 - \frac{\arcsin x}{x} \right)}{x \arcsin x} \quad \left(\frac{0}{0} \text{ form} \right)$$

$$\stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{\frac{1}{\sqrt{1-x^2}} \cdot x - 1 \cdot \arcsin x}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{x - \sqrt{1-x^2} \cdot \arcsin x}{\sqrt{1-x^2} \cdot x^2} = \lim_{x \rightarrow 0} \frac{x - \sqrt{1-x^2} \arcsin x}{x^2}$$

$\nearrow \frac{0}{0}$ form

$$\stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{1 - \frac{-2x}{\sqrt{1-x^2}} \cdot \arcsin x - \sqrt{1-x^2} \cdot \frac{1}{1-x^2}}{2x}$$

$$= \lim_{x \rightarrow 0} \frac{1 + \frac{2x \arcsin x}{\sqrt{1-x^2}} - 1}{2x}$$

$$= \lim_{x \rightarrow 0} \frac{2x \arcsin x}{2x \cdot \sqrt{1-x^2}} = 0$$

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Solution 4

$\lim_{x \rightarrow 0} \left(\frac{1}{\arcsin x} - \frac{1}{x} \right)$ $[\infty - \infty]$ form

(equate denominators)

$= \lim_{x \rightarrow 0} \frac{x - \arcsin x}{x \cdot \arcsin x}$ $\left[\frac{0}{0} \text{ form} \right]$

$\stackrel{LH}{=} \lim_{x \rightarrow 0} \frac{1 - \frac{1}{\sqrt{1-x^2}}}{1 \cdot \arcsin x + x \cdot \frac{1}{\sqrt{1-x^2}}}$

(equate denominators on the upper & lower side)

$= \lim_{x \rightarrow 0} \frac{\frac{\sqrt{1-x^2} - 1}{\sqrt{1-x^2}}}{\frac{\sqrt{1-x^2} \arcsin x + x}{\sqrt{1-x^2}}} = \lim_{x \rightarrow 0} \frac{\sqrt{1-x^2} - 1}{\sqrt{1-x^2} \arcsin x + x}$ $\left[\frac{0}{0} \text{ form} \right]$

(instead of L'Hopital's multiply & divide by $\sqrt{1-x^2} + 1$)

$= \lim_{x \rightarrow 0} \frac{1 - x^2 - 1}{(\sqrt{1-x^2} \arcsin x + x)(\sqrt{1-x^2} + 1)}$
 take this part in x parenthesis

$= \lim_{x \rightarrow 0} \frac{-x \cdot x}{x \left(\frac{\sqrt{1-x^2}}{1} \cdot \frac{\arcsin x}{x} + 1 \right) \left(\frac{\sqrt{1-x^2}}{1} + 1 \right)} = 0$

$\left(\lim_{x \rightarrow 0} \frac{\arcsin x}{x} = 1 \right)$