Math 119 – Quiz #2 – Group T1

Q: Let f be a differentible function on $\mathbb{R}$ such that $f'(x) < 1$

$\forall x > 0$.

If $f(0) = 0$, show that $f(x) < x \quad \forall x > 0$.

<u>Solution;</u> Conside the interval $[0, x]$.

Since f is differantible on $\mathbb{R}$,

- f is continuous on $[0, x]$, and
- f is differentible on $(0, x)$.

Therefore, by the Mean Value Theorem,

$$\exists\, x_0 \in (0, x) : \quad f'(x_0) = \frac{f(x) - f(0)}{x - 0} = \frac{f(x)}{x}.$$

But since $f'(x) < 1 \ \forall x > 0$, $\dfrac{f(x)}{x} < 1 \ \forall x > 0$

$\Rightarrow f(x) < x \ \forall x > 0$ as required.

Alternatively you can define $g(x) = f(x) - x$ and obtain the required conclusion by showing that g is decreasing $\forall x > 0$