Q-4 F-4

Q/ Find $\frac{d}{dx} F(\sqrt{x})$ if $F(x) = \int_5^{\tan(x^2)} \sqrt{1+t^2}\, dt$

Sol^n

1. way

$$F(x) = \int_5^{\tan(x^2)} \sqrt{1+t^2}\, dt \Rightarrow F(\sqrt{x}) = \int_5^{\tan x} \sqrt{1+t^2}\, dt$$

$$\Rightarrow \frac{d\, F(\sqrt{x})}{dx} = \frac{d}{dx} \int_5^{\tan x} \sqrt{1+t^2}\, dt = \sqrt{\underbrace{1+\tan^2 x}_{\sec^2 x}} \cdot \sec^2 x$$

$$= \sec x \cdot \sec^2 x = \underline{\sec^3 x}$$

2. way

$$\frac{d\, F(\sqrt{x})}{dx} = \frac{d\sqrt{x}}{dx} \cdot \frac{d\, F(\sqrt{x})}{d\sqrt{x}} = \frac{d\sqrt{x}}{dx} \cdot \frac{d\, F(u)}{du} = \frac{1}{2\sqrt{x}} \cdot \frac{d}{du} F(u)$$

let $\sqrt{x} = u$

$$F(u) = \int_5^{\tan(u^2)} \sqrt{1+t^2}\, dt$$

$$\Rightarrow \frac{d}{du} F(u) = \frac{d}{dx} \int_5^{\tan(u^2)} \sqrt{1+t^2}\, dt$$

$$= \sqrt{\underbrace{1+\tan^2(u^2)}_{\sec^2(u^2)}} \cdot \sec^2(u^2) \cdot 2u \qquad \text{(by FTC)}$$

$$= \sec^3(u^2) \cdot 2u = \sec^3(x) \cdot 2\sqrt{x}$$

$$\Rightarrow \frac{d\, F(\sqrt{x})}{dx} = \frac{1}{2\sqrt{x}} \cdot \sec^3(x)\, 2\sqrt{x} = \underline{\sec^3(x)}$$

Common mistake

$$\frac{d}{dx} F(\sqrt{x}) \neq F'(\sqrt{x}) \quad \text{because} \quad F'(\sqrt{x}) = \frac{dF(x)}{dx}\Big|_{x=\sqrt{x}}$$

$$\left\{ \begin{array}{l} \text{means put } \sqrt{x} \text{ instead of } x \\ \text{in } F(x) \text{ then take the} \\ \text{derivative of } F(\sqrt{x}) \end{array} \right\} \neq \left\{ \begin{array}{l} \text{means take the derivative} \\ \text{of } F(x) \text{ then put} \\ \sqrt{x} \text{ instead of } x \end{array} \right.$$