Quiz 2

Let k be a non-zero constant and $f(x) = \begin{cases} \frac{x^2}{\sin(kx)} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$

Suppose that $f'(0) = 2$. Find k.

Solution

$$2 = f'(0) = \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0} \frac{\frac{x^2}{\sin(kx)} - 0}{x - 0}$$

$$= \lim_{\substack{x \to 0 \\ (x \neq 0)}} \frac{\cancel{x^2}\,x}{\cancel{x} \cdot \sin(kx)} = \lim_{x \to 0} \left(\frac{kx}{\sin(kx)} \cdot \frac{1}{k} \right) = \frac{1}{k}$$

since $\lim_{x \to 0} \frac{kx}{\sin(kx)} = 1$

$$\Rightarrow k = \frac{1}{2}$$

OR

$$2 = f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{\frac{h^2}{\sin(kh)} - 0}{h}$$

$$= \lim_{\substack{h \to 0 \\ (h \neq 0)}} \frac{\cancel{h^2}\,h}{\cancel{h} \cdot \sin(kh)} = \lim_{h \to 0} \left(\frac{kh}{\sin(kh)} \cdot \frac{1}{k} \right) = \frac{1}{k}$$

since $\lim_{h \to 0} \frac{kh}{\sin(kh)} = 1$

$$\Rightarrow k = \frac{1}{2}$$