

Quiz 3

Show that $f(x) = 3^x + \arctan(x^3)$ is an invertible function. Find $(f^{-1})'(1)$.

$$f'(x) = 3^x \ln 3 + \frac{3x^2}{1+x^6} > 0 \Rightarrow f'(x) > 0 \quad \text{for all } x \text{ in } \mathbb{R}$$

$\overline{>0} \quad \overline{>0} \quad \overline{\geq 0}$

Since f is the sum of two differentiable functions (exponential and inverse tangent), f is differentiable everywhere. Since $f'(x) > 0$, f is increasing. Therefore f is 1-1 and invertible.

$$f(a) = 1 \Rightarrow 3^a + \arctan(a^3) = 1 \Rightarrow a = 0$$

$$\Rightarrow f(0) = 1 \Rightarrow f^{-1}(1) = 0 \quad \text{and}$$

$$f'(0) = 3^0 \cdot \ln 3 + \arctan(0^3) = \ln 3 = f'(0)$$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

$$\Rightarrow (f^{-1})'(1) = \frac{1}{f'(f^{-1}(1))} = \frac{1}{f'(0)} = \frac{1}{\ln 3}$$