Math 119 F3 quiz 2

Show your work and explain your claims.

Let $f(x)$ be a differentiable function on $\mathbb{R}$ with $f'(x) \geq 2x + 1$ for all x, and $f(0) = -2$. Show that $f(2) \geq 0$.

$f(x)$ is diffble everywhere $\Rightarrow$ it is continuous everywhere

So:

* f is cont. on $[0,2]$
* f is diffble on $(0,2)$

(Then Mean Value theorem is applicable)

By MVT there exists $c \in (0,2)$ satisfying

$0 < c < 2$

$$f'(c) = \frac{f(2) - f(0)}{2 - 0}$$

$f'(x) \geq 2x+1$ (given)

then

$f'(c) \geq 2c+1$

$$\Rightarrow f'(c) = \frac{f(2) - (-2)}{2}$$

$$\Rightarrow \frac{f(2)+2}{2} \geq 2c+1 \Rightarrow f(2)+2 \geq 4c+2$$

$$f(2) \geq 4c > 0$$

$\square$