MATH 119F-2 QUIZ 5 (25.12.2020)

Show your work and explain your claims.

Find the limit $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\left(4-\frac{3i}{n}\right)^3$ by interpreting it as a definite integral.

1st way

Consider $\sum_{i=1}^{n} \underbrace{\frac{3}{n}}_{\Delta x}\underbrace{\left(4-\frac{3i}{n}\right)^3}_{f(x_i^*)} = \sum_{i=1}^{n} f(x_i^*)\Delta x_i = \int_a^b f(x)\,dx$

set : Δx $\quad f(x_i^*)$

(riemann sum with equal length subintervals) $\Delta x_i = \Delta x$

$\Delta x = \frac{b-a}{n} = \frac{3}{n} \Rightarrow b-a=3$
$b=a+3$

$\begin{pmatrix} a = x_0 \\ b = x_n \end{pmatrix}$

$\Rightarrow$ $x_1 = x_0 + \Delta x = x_0 + 3/n$
$x_2 = x_1 + \Delta x = x_0 + 2\cdot 3/n$
$\vdots$
$x_i = x_0 + i\,3/n = x_0 + \frac{3i}{n}$

Also let $x_i^* \in [x_{i-1}, x_i]$ to be x_i. $\quad x_i^* = x_i = x_0 + \frac{3i}{n}$

$f(x_i^*) = \left(4-\frac{3i}{n}\right)^3 \Rightarrow f(x_i) = f\left(x_0 + \frac{3i}{n}\right) = \left(4-\frac{3i}{n}\right)^3$

Choose $x_0 = 0$ (so $a=0$ then $b=3$)

then $f\left(\frac{3i}{n}\right) = \left(4-\frac{3i}{n}\right)^3$

$\Rightarrow f(x) = (4-x)^3$

So $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\left(4-\frac{3i}{n}\right)^3 = \int_0^3 (4-x)^3\,dx = \int_0^3 (64-48x+12x^2-x^3)\,dx = \left(64x-24x^2+4x^3-\frac{x^4}{4}\right)\Big|_0^3$

$= 192-216+108-\frac{81}{4} = \frac{255}{4}$

OR

$\int_0^3 (4-x)^3\,dx = -\int_4^1 u^3\,du = -\frac{u^4}{4}\Big|_4^1$

Substitute
$4-x=u$
$-dx=du$

$= \left(-\frac{1}{4}\right) - \left(-\frac{256}{4}\right) = \frac{255}{4}$

$x=0 \Rightarrow u=4$
$x=3 \Rightarrow u=1$

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2nd way

Show your work and explain your claims.

Find the limit $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\left(4 - \frac{3i}{n}\right)^3$ by interpreting it as a definite integral.

Consider $\sum_{i=1}^{n} \underbrace{\frac{3}{n}}_{\Delta x} \underbrace{\left(4 - \frac{3i}{n}\right)^3}_{f(x_i^*)} = \sum_{i=1}^{n} f(x_i^*)\Delta x_i = \int_a^b f(x)\,dx$

set : Δx

(riemann sum with equal legth subintervals) $\Delta x_i = \Delta x$

$\Delta x = \frac{b-a}{n} = \frac{3}{n} \Rightarrow b-a = 3$
$b = a+3$

$\begin{pmatrix} a = x_0 \\ b = x_n \end{pmatrix}$

$\Rightarrow$ $x_1 = x_0 + \Delta x = x_0 + 3/n$
$x_2 = x_1 + \Delta x = x_0 + 2 \cdot 3/n$
$\vdots$
$x_i = x_0 + i\,3/n = x_0 + \frac{3i}{n}$

Also let $x_i^* \in [x_{i-1}, x_i]$ to be x_i. $x_i^* = x_i = x_0 + \frac{3i}{n}$

$f(x_i^*) = \left(4 - \frac{3i}{n}\right)^3 \Rightarrow f(x_i) = f\left(x_0 + \frac{3i}{n}\right) = \left(4 - \frac{3i}{n}\right)^3 = \left(-\left(-4 + \frac{3i}{n}\right)\right)^3 = -\left(-4 + \frac{3i}{n}\right)^3$

$f\left(x_0 + \frac{3i}{n}\right) = -\left(-4 + \frac{3i}{n}\right)^3$

Choose $x_0 = -4$ ($x_0 = a = -4$ then $b = -4+3 = -1$)

Then $f\left(-4 + \frac{3i}{n}\right) = -\left(-4 + \frac{3i}{n}\right)^3 \Rightarrow f(x) = -x^3$
(b)

So $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\left(4 - \frac{3i}{n}\right)^3 = \int_{-4}^{-1} -x^3\,dx = -\frac{x^4}{4}\Big|_{-4}^{-1} = \left(-\frac{1}{4}\right) - \left(-\frac{256}{4}\right) = \frac{255}{4}$
(a)

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3rd way

Show your work and explain your claims.

Find the limit $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}(4 - \frac{3i}{n})^3$ by interpreting it as a definite integral.

Consider $\sum_{i=1}^{n} \underbrace{\frac{1}{n}}_{\Delta x} \underbrace{3\left(4 - \frac{3i}{n}\right)^3}_{f(x_i^*)} = \sum_{i=1}^{n} f(x_i^*)\Delta x_i = \int_a^b f(x)dx$

Set :

(riemann sum with equal length subintervals) $\Delta x_i = \Delta x$

$\Delta x = \frac{b-a}{n} = \frac{1}{n} \Rightarrow b-a = 1$

$b = a+1$

$\left(\begin{matrix} a = x_0 \\ b = x_n \end{matrix}\right)$

$x_1 = x_0 + \Delta x = x_0 + 1/n$

$\Rightarrow x_2 = x_1 + \Delta x = x_0 + 2 \cdot 1/n$

$\vdots$

$x_i = x_0 + i\, 3/n = x_0 + \frac{i}{n}$

Also let $x_i^* \in [x_{i-1}, x_i]$ to be x_i. $x_i^* = x_i = x_0 + \frac{i}{n}$

$f(x_i^*) = 3\left(4 - \frac{i}{n}\right)^3 \Rightarrow f(x_i) = f(x_0 + \frac{i}{n}) = 3\left(4 - 3\frac{i}{n}\right)^3$

Choose $x_0 = 0$ (so $a = 0$ then $b = 1$)

then $f(\frac{i}{n}) = 3\left(4 - 3\frac{i}{n}\right)^3$

$\Rightarrow f(x) = 3(4-3x)^3$

So $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\left(4 - \frac{3i}{n}\right)^3 = \int_0^1 \underbrace{3}_{-du}\underbrace{(4-3x)^3}_{u^3}dx = -\int_4^1 u^3 du = -\frac{u^4}{4}\Big|_4^1 = \left(-\frac{1}{4}\right) - \left(-\frac{256}{4}\right) = \frac{255}{4}$

Substitute

$4 - 3x = u \Rightarrow -3dx = du$

$3dx = -du$

$x = 0 \Rightarrow u = 4$

$x = 1 \Rightarrow u = 1$

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4th way

Show your work and explain your claims.

Find the limit $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}(4-\frac{3i}{n})^3$ by <u>interpreting it as a definite integral.</u>

Consider $\sum_{i=1}^{n}\left(\underbrace{\frac{-3}{n}}_{\text{set : }\Delta x}\underbrace{\left(4-\frac{3i}{n}\right)^3}_{f(x_i^*)}\right) = \sum_{i=1}^{n} f(x_i^*)\Delta x_i = \int_a^b f(x)\,dx$

(riemann sum with equal length subintervals $\Delta x_i = \Delta x$)

$\Delta x = \frac{b-a}{n} = \frac{-3}{n} \Rightarrow b-a=-3$, $b = a-3$

$\begin{pmatrix} a = x_0 \\ b = x_n \end{pmatrix}$

$\Rightarrow x_1 = x_0 + \Delta x = x_0 - 3/n$
$x_2 = x_1 + \Delta x = x_0 - 2\cdot 3/n$
$\vdots$
$x_i = x_0 - i\,3/n = x_0 - \frac{3i}{n}$

Also let $x_i^* \in [x_{i-1}, x_i]$ to be x_i. $x_i^* = x_i = x_0 - \frac{3i}{n}$

$f(x_i^*) = -\left(4-\frac{3i}{n}\right)^3 \Rightarrow f(x_i) = f\left(x_0 - \frac{3i}{n}\right) = -\left(4-\frac{3i}{n}\right)^3$

Choose $x_0 = 4$ ($x_0 = a = 4$ then $b = 4-3 = 1$)

Then $f\left(4-\frac{3i}{n}\right) = \left(4-\frac{3i}{n}\right)^3 \Rightarrow f(x) = -x^3$

So $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\left(4-\frac{3i}{n}\right)^3 = \int_{4\,(a)}^{1\,(b)} -x^3\,dx = -\frac{x^4}{4}\Big|_4^1 = \left(-\frac{1}{4}\right) - \left(-\frac{256}{4}\right) = \frac{255}{4}$

$\Delta x = \frac{3}{n}$

$x_0 = 4 \Rightarrow \quad x_i = 4 + \frac{3i}{n} \quad \& \quad x_n = 7$

try to find this inside f

$f(x_i^*) = \left(4 - \frac{3i}{n}\right)^3$

$= \left(4 - \left(\frac{3i}{n} + 4 - 4\right)\right)$

$= \left(8 - \left(\frac{3i}{n} + 4\right)\right)^3$

given limit $= \int_4^7 (8 - x)^3 \, dx$

$\left(8 - x = u \Rightarrow -dx = du\right) \rightarrow dx \rightarrow -du$

$x = 4 \Rightarrow u = 4$

$x = 7 \Rightarrow u = 1$

$= \int_4^1 -u^3 \, du = -\frac{u^4}{4}\Big|_4^1 = \left(-\frac{1}{4}\right) - \left(-\frac{256}{4}\right) = \frac{255}{4}$

5th soln's sketch

As you see there are many correct solution for this question. But you should write it correctly