Quiz 4 (F-2)

Q: Find the absolute maximum and minimum values of $f(x) = (1-x^2)^{4/5}$ on the interval $[-2, 2]$ if they exist.

S: Since f is continuous and the interval $[-2,2]$ is closed and bounded, by the Extreme Value Theorem f attains the absolute maximum and minimum values.

$$f'(x) = \frac{4}{5} \cdot (1-x^2)^{-1/5} \cdot (-2x) = \frac{4}{5} \frac{-2x}{(1-x^2)^{1/5}}$$

for the critical points, $f'(x) = 0 \Rightarrow x = 0$

for the singular points, $f'(x)$ DNE $\Rightarrow x = \mp 1$

for the endpoints, we check $f(-2), f(2)$

x	-2		-1		0		1		2
$f'(x)$	▨	$-$	0	$+$	0	$-$	0	$+$	▨
$f(x)$		↘	local min	↗	local max	↘	local min	↗	local max

$\Rightarrow$ $f(-2) = (-3)^{4/5} = 3^{4/5}$, $f(2) = (-3)^{4/5} = 3^{4/5}$ } the abs maximum

$f(-1) = 0$, $f(1) = 0$ } the abs minimum

$f(0) = 1$