F1 / Q5

Find $F'(1)$ if $F(x) = \int_5^{5x} x^2 e^{\sqrt{t}}\, dt$

(1) $F(x) = x^2 \int_5^{5x} e^{\sqrt{t}}\, dt$ ((since x^2 is a constant in the integral)

$$\Rightarrow F'(x) = 2x \int_5^{5x} e^{\sqrt{t}}\, dt + x^2 \frac{d}{dx}\left(\int_5^{5x} e^{\sqrt{t}}\, dt \right)$$

$$= 2x \int_5^{5x} e^{\sqrt{t}}\, dt + x^2 . \underbrace{e^{\sqrt{5x}}\ 5.}_{FTC}$$

$$\Rightarrow F'(1) = 2 \underbrace{\int_5^{5} e^{\sqrt{t}}\, dt}_{=0} + e^{\sqrt{5}} . 5$$

$$= 5 e^{\sqrt{5}}$$

OR

(2) $\int_5^{5x} x^2 e^{\sqrt{t}}\, dt = F(x)$ let $\sqrt{t} = a$, ie, $t = a^2$

$\Rightarrow dt = 2a\, da$

$$\Rightarrow \int_5^{5x} x^2 e^{\sqrt{t}}\, dt = \int_{\sqrt{5}}^{\sqrt{5x}} x^2 . e^{a}\, 2a\, da = F(x),$$

Let $a = u$ and $e^a \, da = dv$

$da = du$ $\qquad$ $e^a = v$

$$\Rightarrow F(x) = 2x^2 \left[a \cdot e^a \Big|_{\sqrt{5}}^{\sqrt{5x}} - \int_{\sqrt{5}}^{\sqrt{5x}} e^a \, da \right]$$

$$\Rightarrow F(x) = 2x^2 \left[a e^a \Big|_{\sqrt{5}}^{\sqrt{5x}} - e^a \Big|_{\sqrt{5}}^{\sqrt{5x}} \right)$$

$$F(x) = 2x^2 \left(e^a (a-1) \Big|_{\sqrt{5}}^{\sqrt{5x}} \right)$$

$$= 2x^2 \left[e^{\sqrt{5x}} (\sqrt{5x} - 1) - e^{\sqrt{5}} (\sqrt{5} - 1) \right]$$

$$\Rightarrow F'(x) = 4x \left[e^{\sqrt{5x}} (\sqrt{5x} - 1) - e^{\sqrt{5}} (\sqrt{5} - 1) \right] +$$

$$2x^2 \left[\frac{5}{2\sqrt{5x}} e^{\sqrt{5x}} (\sqrt{5x} - 1) + e^{\sqrt{5x}} \left(\frac{5}{2\sqrt{5x}} \right) \right]$$

$$\Rightarrow F'(1) = 5 e^{\sqrt{5}}$$